PruQuest

MATHEMATICS

8

PruQuest

MATHEMATICS

8

Authors

MEHAK CHAWLA NAGPAL
M.Sc. (Mathematics-Delhi University), B.Ed.

SHWETA SETH
M.Sc. (Mathematics-Delhi University), B.Ed.

TARANJOT KAUR
M.Sc. (Mathematics), B.Ed.

HOLY FAITH INTERNATIONAL (P) LTD.

Updated Edition

Published by
Manik Juneja

HOLY FAITH INTERNATIONAL (P) LTD
Gulab Bhawan 6, Bahadur Shah Zafar Marg
New Delhi-110 002, India

Printed at: **M. GULAB SINGH & SONS (P) LTD.**
B–5/14, Site IV, Industrial Area,
Sahibabad (U.P.)

H0310A5934

PREFACE

Mathematics is widely used in our daily lives in various ways, which makes its study essential. The study of mathematics has to be focused on understanding the concepts and practising their applications to help cultivate reasoning, logic and thinking skills.

The PruQuest Mathematics series aims at the following:

- **Exploration** to emphasize on the discovery approach, that is, building knowledge by doing.
- **Understanding** to help learners actively build new knowledge from experience and prior knowledge, facilitating them to think, reason, analyse and articulate logically.
- **Application** to enable learners to structure the reality around them and prepare them better to encounter a wide variety of problems in life.

The unique approach of the series addresses all the demands of the National Curriculum Framework. It also caters to the needs of different types of learners (Visual, Auditory and Kinaesthetic; VAK).

Key Features

- *Recall* to recapitulate the concepts which help build a chapter
- *Some Examples* to provide students with problem-solving strategies which would further enable them to tackle problems independently
- *Practice Exercises and Comprehensive Exercise* to apply the concepts learnt to problems and real-life situations
- *Enrichment* to enhance the conceptual knowledge
- *HOTS* to enable a learner think deeply about various mathematical ideas and develop mathematical skills such as reasoning and analytical thinking
- *Projects* to helps relate mathematics to everyday life and develop life skills
- *MCQs* to provide extra practice in the form of multiple-choice questions and prepare for different examinations
- *Chapter Check-Up* to provide definitions, key points and formulae to remember
- *Periodic Assessments and Sample Tests* to help in continuous evaluation at regular intervals and assess child's overall understanding

The series offers simple, wide-ranging and interesting learning material and thus aids in overall and complete development of the learner.

UNIT 1 – NUMBER SYSTEM

Chapter 1: Rational Numbers

Rational Numbers, Properties of Rational Numbers: Closure property, Commutative Property, Associative Property for addition, Subtraction, Multiplication and Division, Distributive Property, Concept of Additive Identity and Multiplicative Identity, Concept of Additive Inverse and Multiplicative Inverse, Representation of Rational Numbers on Number Line, Finding Rational Numbers Between Two Given Rational Numbers, Use of Mean to Find Rational Numbers between two Given Rational Numbers, Word Problems on Rational Numbers.

Chapter 2: Powers & Exponents

Exponential Notation of Rational Numbers, Reciprocals, Rational Number with Negative Exponent, Expanded Form using Exponents, Laws of Exponents (with Integral Powers, integral and rational base), Expressing Large and Small Numbers in Scientific Form Using Exponents

Chapter 3: Squares and Square Roots

Square Numbers, Properties of Square Numbers, Patterns in Square Numbers, Some more Patterns, Finding Square of a Number by Column Method, Diagonal Method and Expansion Method, Squares of Rational Numbers, Some Patterns in Finding Squares, Pythagorean Triplet, Square root, Finding Square Root and Perfect Squares, Finding Square root by Division Method, Square Roots of Decimals, Approximate square roots, Estimating The Square Root

Chapter 4: Cubes and Cube Roots

Properties of Cubes, Cubes of Rational Numbers, Checking for Perfect Cube, Cube Root of a Number, Methods of Finding Cube Root of a Number: Using Prime Factorisation Method; Successive Subtraction Method; Estimation Method

Chapter 5: Playing With Numbers

Generalised Form of Numbers, Games with Numbers – Two Digit and Three Digit Numbers, Puzzles Involving Replacement of Alphabets by Digits, Magic Squares, Tests of Divisibility

UNIT 2 – ALGEBRA

Chapter 6: Algebraic Expressions and Identities

Terms, factors and coefficients, like and unlike terms, polynomial and its types, addition and subtraction of algebraic expressions, multiplication of various types of algebraic expressions, division of various types of algebraic expressions, algebraic identities, application of identities

Chapter 7: Factorisation

Factorisation of algebraic expressions, Factorisation by Taking out Common Terms, Factorisation by grouping of terms, Factorisation using Identities, Factorisation by Splitting Middle Term

Chapter 8: Linear Equations in One Variable

Root of a Linear Equation, Solve linear equations in one variable (having variable on one side) and its applications, Solve linear equations in one variable (having variable on both sides) and its applications, reducing equations to simpler form and its applications, equations reducible to linear form and its applications

UNIT 3 – COMMERCIAL MATHS

Chapter 9: Comparing Quantities

Applications of Percentage, Concept of Profit and Loss, Concept of Overhead Expenses, Concept of Discount, Successive Discounts, Concept of Sales Tax and Value Added Tax (VAT), Concept of Compound Interest, Difference Between Simple Interest and Compound Interest, Interest Compounded Half Yearly or Quarterly, Interest Compounded Annually and Rate of interest are Different for Different Years, Interest Compounded Annually and Time is in Fraction of year, Application of Compound Interest Formula, Concept of Growth and Depreciation, Concept of Time and Work, Concept of Taps and Cisterns.

Chapter 10: Direct and Inverse Proportion

Direct proportion and its applications, inverse proportion and its applications

UNIT 4 – GEOMETRY

Chapter 11: Understanding Quadrilaterals

Polygons: Convex and Concave, Classification of Polygons, Regular and Irregular Polygons, Angle Sum Property of a Quadrilateral, Sum of Interior Angles and Exterior Angles of a Polygon, Kinds of quadrilaterals: Trapezium, Isosceles Trapezium, Kite, Parallelogram, Properties of Parallelogram, Some Special Types of Parallelograms: Rectangle, Rhombus, Square

Chapter 12: Practical Geometry

Constructing a quadrilateral: when four sides and one diagonal are given, when three sides and two diagonals are given, when two adjacent sides and three angles are given, when three sides and two included angles are given, when four sides and one angle is given, some special cases of quadrilateral

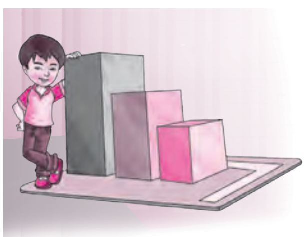
Chapter 13: Visualising Solid Shapes

Different views of 3-D objects namely front, Side and Top View, Maps of Space Around us, Recall Faces, Edges and Vertices, Polyhedrons and Their vertices, Edges and Faces, Regular Polyhedron, Convex Polyhedron, Prisms and Pyramids, Euler's formula for polyhedrons

UNIT 5 – MENSURATION

Chapter 14: Areas and Volumes

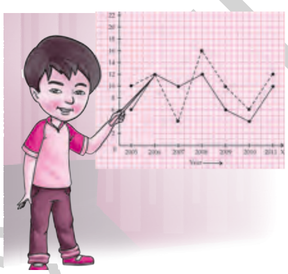
Area of Trapezium, Area of Rhombus, Area of General Quadrilateral, Area of Special Quadrilaterals, Area of Polygon, Solid Shapes, Surface area of Cube, Cuboid and Cylinder, Volume of Cube, Cuboid and Cylinder, Volume and Capacity



UNIT 6 – STATISTICS AND PROBABILITY

Chapter 15: Data Handling

Revisiting Pictograph, Bar Graph and Double Bar Graph, Organising Data, Frequency, Grouping Data, Grouped Frequency Distribution, Bars with a Difference, Circle Graph or Pie Chart, Drawing Pie Charts, Chance and Probability, Random Experiment, Equally Likely Outcomes, Linking Chances of Probability, Outcomes as Events, Chance and Probability in Real Life



UNIT 7 – GRAPHS

Chapter 16: Introduction to Graphs

Bar graph, Pie graph or Circle graph, Histogram, Line graph, Linear Graphs, Application of Graphs

CONTENTS

UNIT 1 – NUMBER SYSTEM 001

1	Rational Numbers	001
2	Powers and Exponents	23
3	Squares and Square Roots	38
4	Cubes and Cube Roots	68
▷	Periodic Assessment 1	83
5	Playing With Numbers	84

UNIT 2 – ALGEBRA 108

6	Algebraic Expressions and Identities	108
7	Factorisation	129
8	Linear Equations in One Variable	149
▷	Periodic Assessment 2	173
▷	Sample Test 1	174

UNIT 3 – COMMERCIAL MATH 176

9	Comparing Quantities	176
10	Direct and Inverse Proportions	212

UNIT 4 – GEOMETRY 226

11	Understanding Quadrilaterals	226
12	Practical Geometry	251
▷	Periodic Assessment 3	268
13	Visualising Solid Shapes	269

UNIT 5 – MENSURATION 287

14 Areas and Volumes 287

UNIT 6 – STATISTICS AND PROBABILITY 321

15 Data Handling 321

UNIT 7 – GRAPHS 337

16 Introduction to Graphs 337

- Periodic Assessment 4 356
- Sample Test 2 357

ANSWERS 359

Chapter 1

Rational Numbers

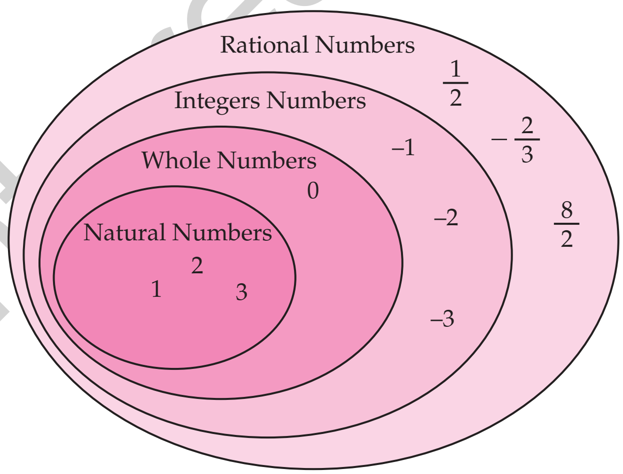
INTRODUCTION

Mathematics is said to be the study of numbers. We started studying mathematics by counting the numbers 1, 2, 3, 4,..., which are called natural numbers. We then studied operations on natural numbers, that is, addition, subtraction, multiplication and division, because of which we felt the need of a new system. Hence, we were introduced to the system of whole numbers, which includes 0 along with the natural numbers. Subtraction of numbers led us to learn negative numbers. Negative numbers combined with whole numbers form the system of integers.

Now, the result of the division of an integer by another non-zero integer may not always be an integer.

For example, $6 \div 3 = 2$, which is an integer, but $3 \div 6$ comes out to be $\frac{3}{6}$ or $\frac{1}{2}$, which is not an integer. Thus, we were introduced to a new system of numbers called rational numbers;

$\frac{3}{4}, \frac{5}{-7}, \frac{-9}{5}, \frac{2}{1}$ are examples of rational numbers.



RECALL

Rational Numbers: A rational number can be defined as the ratio of two integers. It is expressed as $\frac{p}{q}$, where p and q are integers and q is not equal to 0.

For example, $\frac{1}{3}, \frac{4}{7}, \frac{-12}{5}$ and $\frac{-3}{-8}$ are rational numbers.

Note that the system of rational numbers includes natural numbers, whole numbers, integers, fractions and fractions carrying the negative sign.

Positive Rational Numbers: A rational number $\frac{p}{q}$, where q is not equal to 0, is said to be positive if both the numerator and the denominator have the same sign; that is, either both are positive or both are negative.

For example, both: $\frac{3}{22}$ and $\frac{-5}{-12}$ are positive rational numbers.

Negative Rational Numbers: A rational number $\frac{p}{q}$, where q is not equal to 0, is said to be negative if the numerator and the denominator have opposite signs; that is, if the numerator is positive, then the denominator should be negative and vice-versa.

For example, $\frac{-5}{7}$ and $\frac{3}{-14}$ are both negative rational numbers.

Equivalent Rational Numbers: Let $\frac{a}{b}$ be a rational number, and let x be a non-zero integer. Then $\frac{a \times x}{b \times x}$ is a rational number equivalent to the rational number $\frac{a}{b}$.

For example, $\frac{-5}{12}$, $\frac{-5 \times 2}{12 \times 2}$ and $\frac{-5 \times (-3)}{12 \times (-3)}$

or $\frac{-5}{12}$, $\frac{-10}{24}$ and $\frac{15}{-36}$ are equivalent rational numbers.

Standard Form of a Rational number: A rational number is said to be in standard form if its denominator is positive and the HCF of its numerator and denominator is 1.

For example, the rational number $\frac{3}{6}$ has 3 as the common factor of both the numerator and the denominator and hence is not in the standard form. Moreover, the rational number $\frac{4}{-7}$ has a negative denominator and hence is not in the standard form.

A few examples of rational numbers in the standard form are $\frac{-1}{2}, \frac{2}{9}, \frac{3}{8}$ and $\frac{-2}{13}$.

Note

(1) When writing a negative rational number we put the negative sign just before the dividing line.

For example, $-\frac{3}{5}, -\frac{9}{10}$ and $-\frac{4}{3}$ are negative rational numbers.

(2) All natural numbers, whole numbers, integers and fractions are rational, but the converse may not be true. For example, $\frac{3}{5}$ is a rational number, but it is not an integer.

(3) The negative of a rational number is also called the opposite of the number.

PROPERTIES OF RATIONAL NUMBERS

In the last grade, we studied certain properties of integers. Let us verify these properties for rational numbers.

PROPERTIES OF ADDITION

CLOSURE PROPERTY OF ADDITION

We know that $3 + 9 = 12$ (a whole number)

$-5 + 3 = -2$ (an integer)

That is, the sum of two whole numbers is a whole number, and sum of two integers is an integer.

This is called the ***closure property of addition***.

Let us now verify the closure property of addition for rational numbers.

We know how to add two rational numbers. Let us add a few pairs.

$$\frac{3}{5} + \frac{(-2)}{4} = \frac{12 + (-10)}{20} = \frac{2}{20} = \frac{1}{10} \text{ (a rational number)}$$

$$\frac{-5}{7} + \frac{(-4)}{3} = \frac{-15 - 28}{21} = \frac{-43}{21} \text{ (a rational number)}$$

We observe that the sum of two rational numbers is again a rational number. Therefore, we can say that ***rational numbers are closed under addition***.

> In general, if a and b are two rational numbers, then $a + b$ is also a rational number.

COMMUTATIVE PROPERTY OF ADDITION

We know that

$$2 + 7 = 7 + 2$$

$$(-11) + (-3) = (-3) + (-11)$$

That is, whole numbers and integers can be added in any order. This is called the commutative property of addition.

Let us take any two rational numbers, say $-\frac{2}{3}$ and $\frac{5}{7}$, and add them in different orders.

$$-\frac{2}{3} + \frac{5}{7} = \frac{-14 + 15}{21} = \frac{1}{21}$$

$$\frac{5}{7} + \left(-\frac{2}{3}\right) = \frac{5}{7} - \frac{2}{3} = \frac{15 - 14}{21} = \frac{1}{21}$$

We observe that $-\frac{2}{3} + \frac{5}{7} = \frac{5}{7} + \left(-\frac{2}{3}\right)$

Similarly we can check $\frac{7}{8} + \frac{3}{11} = \frac{3}{11} + \frac{7}{8}$

> In general, if a and b are two rational numbers, then $a + b = b + a$.

Hence, we conclude that rational numbers can be added in any order, that is ***addition is commutative for rational numbers.***

ASSOCIATIVE PROPERTY OF ADDITION

We know that $3 + (5 + 7) = (3 + 5) + 7$

$$-2 + [3 + (-4)] = [(-2) + 3] + (-4)$$

That is, whole numbers and integers can be added by pairing them in different ways. This is called the ***associative property of addition***. Let us check this property for rational numbers.

Let us consider any three rational numbers and add them by grouping them in different orders.

Let us consider the three rational numbers $\frac{1}{12}, \frac{3}{4}$ and $-\frac{1}{2}$.

Now, $$\frac{1}{12} + \left\{\frac{3}{4} + \left(-\frac{1}{2}\right)\right\} = \frac{1}{12} + \left(\frac{3-2}{4}\right) = \frac{1}{12} + \frac{1}{4} = \frac{1+3}{12} = \frac{4}{12} = \frac{1}{3}$$

And $$\left(\frac{1}{12} + \frac{3}{4}\right) + \left(-\frac{1}{2}\right) = \left(\frac{1+9}{12}\right) + \left(-\frac{1}{2}\right) = \frac{10}{12} - \frac{1}{2} = \frac{10-6}{12} = \frac{4}{12} = \frac{1}{3}$$

We observe that $$\frac{1}{12} + \left\{\frac{3}{4} + \left(-\frac{1}{2}\right)\right\} = \left(\frac{1}{12} + \frac{3}{4}\right) + \left(-\frac{1}{2}\right)$$

In general if, a, b, c, are three rational numbers, then $a + (b + c) = (a + b) + c$.

Thus, ***addition is associative for rational numbers***.

PROPERTY OF ZERO (ADDITIVE IDENTITY)

The number 0 is quite significant in mathematics.

Let us look at the following:

$$-7 + 0 = -7,\ -\frac{5}{9} + 0 = -\frac{5}{9}$$

We observe that when zero is added to any rational number, the result obtained is the same number.

Thus, ***zero is the additive identity for rational numbers.***

In general, if a is any rational number, then $a + 0 = 0 + a = a$

Note

Zero is the additive identity for whole numbers and integers as well.

NEGATIVE OF A NUMBER (EXISTENCE OF ADDITIVE INVERSE)

When we study integers, we come across positive integers and negative integers. An integer when added to its negative or opposite gives zero.

Note

The additive inverse of zero is zero.

For example, $3 + (-3) = 0 = (-3) + (3)$

Here, –3 is the negative of 3, as the sign has been changed. Similarly, 3 is the negative of –3.

For rational numbers

$$\frac{1}{3} + \left(-\frac{1}{3}\right) = \frac{1+(-1)}{3} = \frac{1-1}{3} = \frac{0}{3} = 0$$

And $$\left(-\frac{1}{3}\right) + \frac{1}{3} = \frac{(-1)+1}{3} = \frac{-1+1}{3} = 0$$

Thus, upon adding $\frac{1}{3}$ and $\left(-\frac{1}{3}\right)$, we get 0.

Here $-\frac{1}{3}$ is said to be negative of $\frac{1}{3}$ and $\frac{1}{3}$ is the negative of $-\frac{1}{3}$.

Here $-\frac{p}{q}$ is called the ***negative of rational number*** $\frac{p}{q}$ ***and*** $\frac{p}{q}$ ***is called the negative of*** $-\frac{p}{q}$.

Mathematically, $-\frac{p}{q}$ is the ***additive inverse*** of $\frac{p}{q}$ and $\frac{p}{q}$ is the additive inverse of $-\frac{p}{q}$.

In general, if $\frac{p}{q}$ is a rational number, then there exists its opposite $-\frac{p}{q}$, such that

$$\frac{p}{q} + \left(\frac{-p}{q}\right) = 0$$

Hence, $-\frac{p}{q}$ and $\frac{p}{q}$ are called opposites or negatives or the additive inverse of each other.

Thus, the additive inverse of $\frac{7}{11}$ is $\frac{-7}{11}$, and the additive inverse of $\frac{-15}{8}$ is $\frac{15}{8}$.

SOME EXAMPLES

Example 1: Verify the closure property of addition for the following rational numbers:

(i) $\frac{4}{7}$ and $\frac{-6}{11}$ (ii) $\frac{3}{4}$ and $\frac{-5}{6}$

Solution: (i) We have $\frac{4}{7} + \frac{(-6)}{11} = \frac{44-42}{77} = \frac{2}{77}$, which is a rational number.

(ii) We have $\frac{3}{4} + \frac{(-5)}{6} = \frac{9-10}{12} = \frac{-1}{12}$, which is a rational number.

Hence, the closure property of addition is verified.

Example 2: Verify the commutative property of addition for $a = \frac{5}{7}$ and $b = \frac{-8}{21}$.

Solution: We have $a + b = \frac{5}{7} + \frac{(-8)}{21} = \frac{15+(-8)}{21} = \frac{7}{21} = \frac{1}{3}$

And $b + a = \frac{(-8)}{21} + \frac{5}{7} = \frac{(-8)+15}{21} = \frac{7}{21} = \frac{1}{3}$

$\therefore$ $a + b = b + a$

Hence, the commutative property of addition is verified.

Example 3: Verify the associative property of addition for the rational numbers $\frac{-2}{3}, \frac{5}{4}, \frac{7}{12}$.

Solution: We have $\frac{-2}{3} + \left(\frac{5}{4} + \frac{7}{12}\right) = \frac{-2}{3} + \frac{15+7}{12} = \frac{-2}{3} + \frac{22}{12} = \frac{-2}{3} + \frac{11}{6} = \frac{-4+11}{6} = \frac{7}{6}$

And $\left(\frac{-2}{3} + \frac{5}{4}\right) + \frac{7}{12} = \left(\frac{-8+15}{12}\right) + \frac{7}{12} = \frac{7}{12} + \frac{7}{12} = \frac{7+7}{12} = \frac{14}{12} = \frac{7}{6}$

$\therefore$ $\frac{-2}{3} + \left(\frac{5}{4} + \frac{7}{12}\right) = \left(\frac{-2}{3} + \frac{5}{4}\right) + \frac{7}{12}$

Example 4: Simplify $\frac{1}{13} + \left(-\frac{3}{4}\right) + \left(-\frac{3}{26}\right) - \frac{5}{16}$.

Solution: We have $\frac{1}{13} + \left(-\frac{3}{4}\right) + \left(-\frac{3}{26}\right) - \frac{5}{16}$

$$= \frac{1}{13} - \frac{3}{4} - \frac{3}{26} - \frac{5}{16} = \frac{16-156-24-65}{208} = \frac{-229}{208}$$

Clearly, this simplification involves lengthy calculations. Let us try to simplify the same expression in another way, using properties.

$$\frac{1}{13} + \left(-\frac{3}{4}\right) + \left(-\frac{3}{26}\right) - \frac{5}{16}$$

$$= \frac{1}{13} + \left(-\frac{3}{26}\right) + \left(-\frac{3}{4}\right) - \frac{5}{16}$$ [Using the commutative property of addition]

$$= \left[\frac{1}{13} + \left(-\frac{3}{26}\right)\right] + \left[-\frac{3}{4} - \frac{5}{16}\right]$$ [Using the associative property of addition]

$$= \left[\frac{2-3}{26}\right] + \left[\frac{-12-5}{16}\right]$$

$$= \frac{-1}{26} + \left[\frac{-17}{16}\right] = \frac{-1}{26} - \frac{17}{16} = \frac{-8 - 221}{208} = \frac{-229}{208}$$

Example 5: Verify that $-(-x) = x$, given that $x = \frac{1}{7}$.

Solution: We have $x = \frac{1}{7}$.

The additive inverse of $x = \frac{1}{7}$ is $-x = -\frac{1}{7}$ because $\frac{1}{7} + \left(-\frac{1}{7}\right) = 0$.

The same equality, that is, $\frac{1}{7} + \left(-\frac{1}{7}\right) = 0$, shows that the additive inverse of $-\frac{1}{7}$ is $\frac{1}{7}$

or $-\left(-\frac{1}{7}\right) = \frac{1}{7}$.

Hence, $-(-x) = x$ is verified.

Example 6: Write the additive inverse of the following:

(i) $-\frac{2}{3}$ (ii) $\frac{5}{8}$

Solution: (i) $-\frac{2}{3} + \left(\frac{2}{3}\right) = 0$

Thus, $\frac{2}{3}$ is the additive inverse of $-\frac{2}{3}$.

(ii) $\frac{5}{8} + \left(-\frac{5}{8}\right) = 0$

Thus, $-\frac{5}{8}$ is the additive inverse of $\frac{5}{8}$.

PROPERTIES OF SUBTRACTION

CLOSURE PROPERTY OF SUBTRACTION

Let us see what happens when we subtract two rational numbers.

$$-\frac{15}{17} - \frac{1}{3} = \frac{-45 - 17}{51} = \frac{-62}{51},\ 0 - \frac{7}{12} = \frac{-7}{12}$$

We observe that the results obtained are also rational numbers.

Hence, we conclude that the difference of two rational numbers is always a rational number; that is, ***rational numbers are closed under subtraction.***

COMMUTATIVE PROPERTY OF SUBTRACTION

Let us take two rational numbers and find their difference in different orders.

$$\frac{4}{11}-\frac{1}{3}=\frac{12-11}{33}=\frac{1}{33}$$

And $$\frac{1}{3}-\frac{4}{11}=\frac{11-12}{33}=\frac{-1}{33}$$

We observe that $$\frac{4}{11}-\frac{1}{3}\neq\frac{1}{3}-\frac{4}{11}$$

Let us look at one more example:

$$\left(-\frac{4}{13}\right)-\frac{2}{5}=\frac{-20-26}{65}=\frac{-46}{65}$$

$$\frac{2}{5}-\left(-\frac{4}{13}\right)=\frac{26-(-20)}{65}=\frac{26+20}{65}=\frac{46}{65}$$

$\therefore$ $$\left(-\frac{4}{11}\right)-\frac{2}{5}\neq\frac{2}{5}-\left(-\frac{4}{13}\right)$$

Thus, we conclude that ***rational numbers are not commutative under subtraction***.

ASSOCIATIVE PROPERTY OF SUBTRACTION

Let us consider three rational numbers, say $\frac{2}{7}, \frac{1}{3}$ and $\frac{5}{6}$.

Now $$\frac{2}{7}-\left(\frac{1}{3}-\frac{5}{6}\right)=\frac{2}{7}-\left(\frac{2-5}{6}\right)=\frac{2}{7}-\frac{-3}{6}=\frac{2}{7}+\frac{3}{6}=\frac{12+21}{42}=\frac{33}{42}$$

And $$\left(\frac{2}{7}-\frac{1}{3}\right)-\frac{5}{6}=\left(\frac{6-7}{21}\right)-\frac{5}{6}=\frac{-1}{21}-\frac{5}{6}=\frac{-2-35}{42}=\frac{-37}{42}$$

So $$\frac{2}{7}-\left(\frac{1}{3}-\frac{5}{6}\right)\neq\left(\frac{2}{7}-\frac{1}{3}\right)-\frac{5}{6}$$

The same can be observed in a few more examples.

Thus, ***subtraction is not associative for rational numbers***.

SOME EXAMPLES

Example 1: Verify the closure property of subtraction for the rational numbers $a=\frac{-9}{11}$ and $b=\frac{3}{7}$.

Solution: We have

$$a-b=\frac{-9}{11}-\frac{3}{7}=\frac{-63-33}{77}=\frac{-96}{77}$$, which is a rational number.

$\therefore$ Closure property of subtraction is verified.

Example 2: (i) Using $x=\frac{2}{3}$ and $y=\frac{7}{5}$ show that the subtraction of rational numbers is not commutative.

(ii) Using $x=\frac{-4}{5}$, $y=\frac{3}{7}$ and $z=\frac{-1}{10}$ show that the subtraction of rational numbers is not associative.

Solution: (i) $x - y = \frac{2}{3} - \frac{7}{5} = \frac{10 - 21}{15} = \frac{-11}{15}$

$y - x = \frac{7}{5} - \frac{2}{3} = \frac{21 - 10}{15} = \frac{11}{15}$

Clearly, $x - y \neq y - x$; therefore, subtraction is not commutative for rational numbers.

(ii) $x - (y - z) = \frac{-4}{5} - \left\{\frac{3}{7} - \frac{(-1)}{10}\right\} = \frac{-4}{5} - \left\{\frac{30 - (-7)}{70}\right\} = \frac{-4}{5} - \frac{30 + 7}{70}$

$= \frac{-4}{5} - \frac{37}{70} = \frac{-56 - 37}{70} = \frac{-93}{70}$

$(x - y) - z = \left(\frac{-4}{5} - \frac{3}{7}\right) - \frac{(-1)}{10} = \frac{-28 - 15}{35} - \frac{(-1)}{10} = \frac{-43}{35} - \frac{(-1)}{10}$

$= \frac{-86 - (-7)}{70} = \frac{-86 + 7}{70} = \frac{-79}{70}$

Clearly, $x - (y - z) \neq (x - y) - z$.

$\therefore$ Subtraction is not associative for rational numbers.

Example 3: What should be added to $\frac{1}{7}$ to get $\frac{5}{12}$?

Solution: Let the required number be x. Then,

$$\frac{1}{7} + x = \frac{5}{12}$$

$$\Rightarrow \quad x = \frac{5}{12} - \frac{1}{7} = \frac{5 \times 7 - 1 \times 12}{84} = \frac{35 - 12}{84} = \frac{23}{84}$$

So, the required rational number is $\frac{23}{84}$.

PRACTICE EXERCISE 1.1

(1) Verify the closure property and the commutative property of addition for each of the following numbers:

(i) $\frac{2}{7}, \frac{1}{9}$ (ii) $-\frac{3}{5}, \frac{4}{9}$ (iii) $\frac{5}{4}, -\frac{2}{5}$

(2) Verify the associative property of addition for the following rational numbers:

(i) $\frac{3}{2}, \frac{1}{4}, \frac{7}{6}$ (ii) $\frac{-2}{5}, \frac{3}{10}, \frac{-1}{15}$ (iii) $\frac{8}{3}, \frac{7}{2}, \frac{-4}{3}$

(3) Write the additive inverse of each of the following rational numbers:

(i) $\frac{3}{4}$ (ii) $-\frac{5}{21}$ (iii) $\frac{-4}{-43}$

(4) Find the property under addition used in each of the following:

(i) $\frac{2}{3} + 0 = 0 + \frac{2}{3} = \frac{2}{3}$ (ii) $\frac{-7}{4} + \frac{5}{3} = \frac{5}{3} + \frac{(-7)}{4}$

(5) Which number should be added to $-\frac{5}{8}$ to get $-\frac{2}{3}$?

(6) Which number should be subtracted from $-\frac{2}{7}$ to get $\frac{11}{63}$?

(7) Using appropriate properties of addition of rational numbers, solve the following:

(i) $\frac{3}{7} + \frac{-3}{8} + \frac{-4}{7} + \frac{-7}{8}$ (ii) $\frac{4}{11} + \frac{5}{7} + \frac{9}{8} + \frac{-20}{11}$ (iii) $\frac{2}{13} + \frac{-3}{8} + \frac{-5}{13} + \frac{-12}{13}$

(8) Show that $-(-x) = x$ for each of the following:

(i) $x = \frac{2}{7}$ (ii) $x = \frac{-3}{7}$

(9) Find the additive inverse of the negative of $\frac{-5}{11}$.

(10) Find the sum of the additive inverse of $\frac{2}{-9}$ and the negative of $\frac{8}{5}$.

(11) A drum full of kerosene weighs $21\frac{2}{3}$ L. If the empty drum weighs $11\frac{3}{5}$ L, then what is the weight of the kerosene in the drum?

(12) Solve each of the following:

(i) $\frac{1}{4} - \frac{-3}{8}$ (ii) $\frac{-6}{7} - \frac{-13}{14}$ (iii) $\frac{8}{11} - \frac{3}{22}$ (iv) $\frac{-13}{15} - \frac{1}{5}$

(13) The sum of two rational numbers is –7. If one of them is $-\frac{7}{6}$, find the other.

(14) Simplify:

(i) $\frac{3}{8} - \frac{-2}{9} + \frac{-5}{36}$ (ii) $\frac{-3}{2} + \frac{5}{4} - \frac{7}{4}$ (iii) $\frac{-11}{5} - \frac{-1}{10} - \frac{-3}{7}$

PROPERTIES OF MULTIPLICATION

CLOSURE PROPERTY OF MULTIPLICATION

Let us now consider the product of two rational numbers.

$$\frac{1}{3} \times \frac{9}{7} = \frac{9}{21} = \frac{3}{7},\ -\frac{2}{7} \times \left(-\frac{9}{13}\right) = \frac{18}{91},\ 0 \times \frac{13}{15} = 0$$

We observe that each of the products above is a rational number.

Hence, the product of two rational numbers is also a rational number; that is ***rational numbers are closed under multiplication.***

In general, if a and b are two rational numbers, then $a \times b$ is also a rational number.

COMMUTATIVE PROPERTY OF MULTIPLICATION

Let us take two rational numbers and multiply them in different orders.

$$\frac{5}{11} \times \frac{4}{3} = \frac{20}{33}$$

And $\frac{4}{3} \times \frac{5}{11} = \frac{20}{33}$

$\therefore$ $\frac{5}{11} \times \frac{4}{3} = \frac{4}{3} \times \frac{5}{11}$

Similarly, we can check that $\frac{1}{2} \times \left(-\frac{3}{8}\right) = -\frac{3}{16} = \left(-\frac{3}{8}\right) \times \frac{1}{2}$

In general, if a and b are two rational numbers, then $a \times b = b \times a$.

Hence, ***rational numbers are commutative under multiplication.***

ASSOCIATIVE PROPERTY OF MULTIPLICATION

Let us consider three rational numbers, say $-\frac{3}{8}, \frac{1}{7}$ and $\frac{2}{5}$.

Now $-\frac{3}{8} \times \left(\frac{1}{7} \times \frac{2}{5}\right) = -\frac{3}{8} \times \left(\frac{2}{35}\right) = -\frac{3}{140}$

And $\left(-\frac{3}{8} \times \frac{1}{7}\right) \times \frac{2}{5} = \left(-\frac{3}{56}\right) \times \frac{2}{5} = -\frac{3}{140}$

Here, we observe that $-\frac{3}{8} \times \left(\frac{1}{7} \times \frac{2}{5}\right) = \left(-\frac{3}{8} \times \frac{1}{7}\right) \times \frac{2}{5}$

The same can be observed if we take a few more examples.

Thus, multiplication is associative for rational numbers.

In general, if a, b and c are three rational numbers, then $a \times (b \times c) = (a \times b) \times c$.

PROPERTY OF 1 (MULTIPLICATIVE IDENTITY)

Let us now look at a few multiplications involving 1.

$$2 \times 1 = 2,\ 0 \times 1 = 0,\ -13 \times 1 = -13,\ -\frac{5}{8} \times 1 = -\frac{5}{8}$$

In the above examples, we observe that when a rational number is multiplied by 1, we get the same rational number. Thus, 1 is the ***multiplicative identity*** of rational numbers.

RECIPROCAL OF A NUMBER (MULTIPLICATIVE INVERSE)

Consider the rational number $\frac{4}{9}$.

By which number should $\frac{4}{9}$ be multiplied so that the product is 1?

We can easily find that $\frac{4}{9} \times \frac{9}{4} = 1$.

Hence, the required rational number by which $\frac{4}{9}$ should be multiplied to get 1 as the product is $\frac{9}{4}$.

Thus, the reciprocal of $\frac{4}{9}$ is $\frac{9}{4}$, and the reciprocal of $\frac{9}{4}$ is $\frac{4}{9}$.

Let us look at one more example.

By which number should we multiply the rational number $-\frac{2}{3}$ to get 1 as the product ?

We find that $\left(-\frac{2}{3}\right)\times\left(-\frac{3}{2}\right) = 1$. Thus, the reciprocal of $-\frac{2}{3}$ is $-\frac{3}{2}$, and the reciprocal of $\frac{-3}{2}$ is $\frac{-2}{3}$.

Hence, there exists a multiplicative inverse $\frac{q}{p}$ for every rational number $\frac{p}{q}$ such that $\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = 1$.

Note

The multiplicative inverse of 0 does not exist, as there is no rational number which when multiplied by 0 gives 1.

Moreover, $\frac{q}{p}$ is called the reciprocal or inverse of $\frac{p}{q}$, and $\frac{p}{q}$ is also called inverse of $\frac{q}{p}$.

Note

(1) The multiplicative inverse of a positive rational number is positive and that of a negative rational number is negative.

(2) The numbers 1 and –1 are their own reciprocals.

DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION FOR RATIONAL NUMBERS

Let us consider the three rational numbers $\frac{3}{2}, -\frac{5}{8}, \frac{2}{13}$.

Now $$\frac{3}{2}\times\left[-\frac{5}{8}+\frac{2}{13}\right] = \frac{3}{2}\times\left[\frac{-65+16}{104}\right] = \frac{3}{2}\times\left[\frac{-49}{104}\right] = -\frac{147}{208} \quad \ldots(i)$$

Moreover, $$\left[\left(\frac{3}{2}\right)\times\left(-\frac{5}{8}\right)\right]+\left[\left(\frac{3}{2}\right)\times\left(\frac{2}{13}\right)\right] = \left[-\frac{15}{16}\right]+\left[\frac{6}{26}\right] = -\frac{15}{16}+\frac{6}{26} = \frac{-195+48}{208} = \frac{-147}{208} \quad \ldots(ii)$$

From (*i*) and (*ii*), we conclude that

$$\frac{3}{2}\times\left[-\frac{5}{8}+\frac{2}{13}\right] = \left[\left(\frac{3}{2}\right)\times\left(-\frac{5}{8}\right)\right]+\left[\left(\frac{3}{2}\right)\times\left(\frac{2}{13}\right)\right]$$

Similarly, we can verify the distributive property of multiplication over subtraction.

In general, if a, b, c are three rational numbers, then

(i) $a\times(b+c) = a\times b + a\times c$ [Distributive property of multiplication over addition]

(ii) $a\times(b-c) = a\times b - a\times c$ [Distributive property of multiplication over subtraction]

SOME EXAMPLES

Example 1: Verify the associative property of multiplication for $\frac{3}{5}, \frac{7}{9}$ and $\frac{5}{7}$.

Solution: We have $\frac{3}{5}\times\left(\frac{7}{9}\times\frac{5}{7}\right) = \frac{3}{5}\times\frac{5}{9} = \frac{1}{3}$

And $\left(\frac{3}{5}\times\frac{7}{9}\right)\times\frac{5}{7} = \frac{7}{15}\times\frac{5}{7} = \frac{1}{3}$

$\therefore$ $\frac{3}{5}\times\left(\frac{7}{9}\times\frac{5}{7}\right) = \left(\frac{3}{5}\times\frac{7}{9}\right)\times\frac{5}{7}$

Hence, the associative property of multiplication is verified.

Example 2 Simplify $\left(-\frac{3}{6}\right) \times \left(\frac{13}{5}\right) \times \left(\frac{8}{20}\right) \times \left(\frac{4}{1300}\right)$.

Solution: We have

$\left(-\frac{3}{6}\right) \times \left(\frac{13}{5}\right) \times \left(\frac{8}{20}\right) \times \left(\frac{4}{1300}\right) = -\frac{3}{6} \times \frac{8}{20} \times \frac{13}{5} \times \frac{4}{1300}$ [Using the commutative property of multiplication]

$= -\frac{1}{625}$ [Cancelling out the common terms]

Note

(1) The concepts of additive identity and multiplicative identity of rational numbers should not be intermixed. For a rational number $\frac{p}{q}$, the additive identity is 0 and the multiplicative identity is 1.

(2) The additive inverse and the multiplicative inverse should be found carefully. For the rational number $\frac{p}{q}$, the additive inverse is $-\frac{p}{q}$, and the multiplicative inverse is $\frac{q}{p}$.

Example 3: Write the multiplicative inverse of the following:

(i) 1 (ii) $-\frac{2}{11}$ (iii) (–1)

Solution: (i) $1 \times 1 = 1$

Thus, 1 is multiplicative inverse of 1.

(ii) $\left(\frac{-2}{11}\right) \times \left(\frac{-11}{2}\right) = 1$

Thus, $\frac{-11}{2}$ is multiplicative inverse of $\frac{-2}{11}$.

(iii) $(-1) \times (-1) = 1$. Thus, (–1) is the multiplicative inverse of (–1).

Example 4: Using suitable properties, evaluate $\frac{4}{8} \times \frac{1}{7} + \frac{3}{8} \times \frac{1}{7}$.

Solution: $\frac{4}{8} \times \frac{1}{7} + \frac{3}{8} \times \frac{1}{7} = \frac{1}{7} \times \frac{4}{8} + \frac{1}{7} \times \frac{3}{8}$ [Using the commutative property of multiplication]

$= \frac{1}{7} \times \left(\frac{4}{8} + \frac{3}{8}\right)$ [Using the distributive property of multiplication over addition]

$= \frac{1}{7} \times \left(\frac{7}{8}\right) = \frac{1}{8}$

Example 5: Using suitable properties, simplify $\frac{4}{9} \times \left(\frac{-3}{7}\right) + \frac{3}{14} + \left(\frac{-3}{7}\right) \times \frac{2}{9}$.

Solution: We have $\frac{4}{9} \times \left(\frac{-3}{7}\right) + \frac{3}{14} + \left(\frac{-3}{7}\right) \times \frac{2}{9}$

$= \left(\frac{-3}{7}\right) \times \frac{4}{9} + \frac{3}{14} + \left(\frac{-3}{7}\right) \times \frac{2}{9}$ [Using the commutative property of multiplication]

$= \left(\frac{-3}{7}\right) \times \frac{4}{9} + \left(\frac{-3}{7}\right) \times \frac{2}{9} + \frac{3}{14}$ [Using the commutative property of addition]

$= \left(\frac{-3}{7}\right) \times \left(\frac{4}{9} + \frac{2}{9}\right) + \frac{3}{14}$ [Using the distributive property of multiplication over addition]

$= \frac{-3}{7} \times \left(\frac{6}{9}\right) + \frac{3}{14} = \frac{-2}{7} + \frac{3}{14} = \frac{-4+3}{14} = \frac{-1}{14}$

PROPERTIES OF DIVISION

CLOSURE PROPERTY OF DIVISION

We see that

$$\frac{2}{5} \div \left(\frac{-1}{3}\right) = \frac{-6}{5}, \ -\frac{3}{4} \div \left(\frac{1}{2}\right) = \frac{-6}{4}$$

Similarly, for $0 \div \left(\frac{1}{9}\right)$, what do we observe ?

The result is 0.

In the above examples, we notice that the result of the division of two rational numbers is a rational number.

Now, let's take

$0 \div \left(\frac{13}{21}\right)$, which is equal to 0, but $\left(\frac{13}{21}\right) \div 0$ is not defined.

Hence, the result of the division of two rational numbers is not always a rational number; that is ***rational numbers are not closed under division.***

COMMUTATIVE PROPERTY OF DIVISION

Let us take two rational numbers and divide them in different orders.

$$\frac{2}{7} \div \frac{3}{4} = \frac{2}{7} \times \frac{4}{3} = \frac{8}{21}$$

$$\frac{3}{4} \div \frac{2}{7} = \frac{3}{4} \times \frac{7}{2} = \frac{21}{8}$$

We observe that $\frac{2}{7} \div \frac{3}{4} \neq \frac{3}{4} \div \frac{2}{7}$

Thus, ***rational numbers are not commutative under division.***

Note

Division by 0 is meaningless.

ASSOCIATIVE PROPERTY OF DIVISION

Let us take the three rational numbers $-\frac{1}{12}, -\frac{1}{6}$ and $\frac{1}{2}$.

Now $-\frac{1}{12} \div \left(-\frac{1}{6} \div \frac{1}{2}\right) = -\frac{1}{12} \div \left(-\frac{1}{3}\right) = \frac{1}{4}$

And $\left\{-\frac{1}{12} \div \left(-\frac{1}{6}\right)\right\} \div \frac{1}{2} = \left(\frac{1}{2}\right) \div \frac{1}{2} = 1$

We observe that $-\frac{1}{12} \div \left(\frac{-1}{6} \div \frac{1}{2}\right) \neq \left\{\frac{-1}{12} \div \left(-\frac{1}{6}\right)\right\} \div \frac{1}{2}$

Thus, we can conclude that ***division is not associative for rational numbers***.

PRACTICE EXERCISE 1.2

(1) Identify the properties used in each of the following:

(i) $\frac{4}{11} \times \frac{-3}{17} = \frac{-3}{17} \times \frac{4}{11}$

(ii) $\frac{-3}{11} \times \left(\frac{4}{3} \times \frac{3}{8}\right) = \left(\frac{-3}{11} \times \frac{4}{3}\right) \times \frac{3}{8}$

(iii) $\frac{9}{11} \times \left(\frac{3}{4} - \frac{2}{6}\right) = \left(\frac{9}{11} \times \frac{3}{4}\right) - \left(\frac{9}{11} \times \frac{2}{6}\right)$

(2) For each of the following, verify the closure property and the commutative property of multiplication:

(i) $\frac{2}{9}, \frac{3}{11}$ (ii) $\frac{5}{8}, \frac{-2}{7}$ (iii) $\frac{-13}{11}, \frac{33}{12}$ (iv) $\frac{-1}{8}, \frac{4}{-9}$

(3) For each of the following, verify the associative property of multiplication:

(i) $\frac{2}{5}, \frac{7}{3}, \frac{-4}{5}$ (ii) $\frac{3}{7}, \frac{-4}{9}, \frac{-11}{7}$ (iii) $\frac{11}{12}, \frac{-17}{3}, \frac{2}{11}$

(4) Write the multiplicative inverse of each of the following rational numbers:

(i) $\frac{2}{7}$ (ii) $\frac{-3}{5}$ (iii) $\frac{8}{-11}$ (iv) $\frac{-4}{-9}$

(5) Multiply $\frac{4}{-7}$ and its additive inverse.

(6) Multiply $\frac{4}{15}$ with its multiplicative inverse.

(7) Add $\frac{-1}{3}$ and its multiplicative inverse.

(8) Verify that $-\frac{4}{5} \times \left(0 - \frac{9}{2}\right) = \left(-\frac{4}{5} \times 0\right) + \left(\frac{4}{5} \times \frac{9}{2}\right)$.

(9) For $a = \frac{2}{7}$ and $b = \frac{-9}{5}$, verify the following

(i) $a + b = b + a$ (ii) $a \times b = b \times a$

(10) For $x = \frac{1}{2}$, $y = \frac{3}{5}$ and $z = \frac{-7}{6}$, verify the following properties and also name each property:

(i) $(x + y) + z = x + (y + z)$ (ii) $(xy)z = x(yz)$ (iii) $x(y - z) = xy - xz$

(11) Using suitable properties of rational numbers, evaluate the following:

(i) $\frac{3}{14} \times \frac{-5}{9} \times \frac{-21}{10} \times \frac{4}{-3}$ (ii) $\frac{-9}{11} \times \frac{13}{8} \times \frac{12}{13} \times \frac{22}{3}$ (iii) $\frac{-3}{2} \times \frac{5}{4} + \frac{-3}{2} \times \frac{-7}{6}$

(12) Simplify each of the following by using suitable properties. Also name each property.

(i) $\left[\frac{1}{2} \times \frac{3}{4}\right] + \left[\frac{1}{2} \times \frac{2}{5}\right]$ (ii) $\left[\frac{2}{5} \times \frac{3}{7}\right] - \left[\frac{2}{5} \times \frac{1}{7}\right]$

(13) The additive inverse of x is same as the multiplicative inverse of $\frac{5}{9}$. Find the value of x.

(14) Check whether $2\frac{1}{5}$ is the multiplicative inverse of 2.2. Is $333\frac{1}{3}$ the multiplicative inverse of 0.003?

(15) Use the distributive property of multiplication of rational numbers over addition or subtraction to simplify the following:

(i) $\frac{1}{7} \times \left[\frac{1}{5} + \frac{6}{5}\right]$ (ii) $\frac{2}{5} \times \left[\frac{1}{7} + \frac{2}{9}\right]$

REPRESENTATION OF RATIONAL NUMBERS ON THE NUMBER LINE

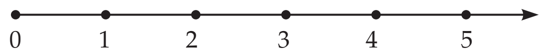
We have earlier studied how to represent natural numbers, whole numbers and integers on the number line. We have also represented rational numbers on the number line in the last grade. Let us recapitulate.

(i) Natural numbers

Each mark represents a unique natural number, and every natural number can be uniquely represented on the number line. The line extends indefinitely only to the right of 1.

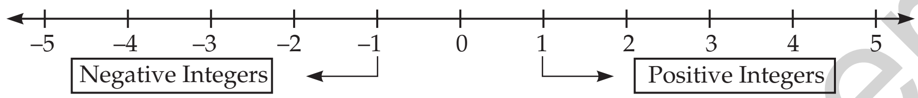
(ii) Whole numbers

Each mark represents a whole number on the number line. At the same time every whole number can be uniquely represented on the number line. The line extends indefinitely to the right, but from 0. There are no numbers to the left of 0 on the number line.



(iii) Integers

For integers a number line extends indefinitely on both sides of 0, as the numbers are distributed on both sides of 0. Negative numbers are marked at equal distances to the left of 0, and positive numbers are marked at equal distances to the right of 0 as shown below.



In the above number lines, there are a few things that are common.

(i) If we look at the representation of natural numbers on the number line and are asked to find out the natural numbers between 1 and 5, we can clearly see that there are three natural numbers between 1 and 5, which are 2, 3 and 4.

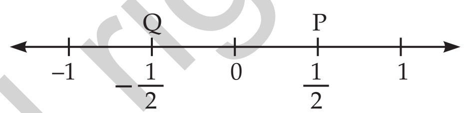
(ii) In a similar manner, if we look at the representation of whole numbers on the number line, we can find the whole numbers lying between 0 and 5, which are 1, 2, 3 and 4.

(iii) Even in integers, we can count the integers that lie between –3 and 2.

Hence, we conclude that in all the above cases, it is possible to find the exact number of numbers lying between any two numbers.

Let us now take a look at the representation of rational numbers on the number line.

(iv) Rational numbers



The line extends indefinitely on both sides of 0. There are no numbers between the integers on the number line, but here we can see that there are numbers between two distinct numbers.

Point P on the number line which lies halfway between 0 and 1 has been labelled $\frac{1}{2}$.

Similarly, point Q which lies exactly midway between 0 and –1 has been labelled $-\frac{1}{2}$.

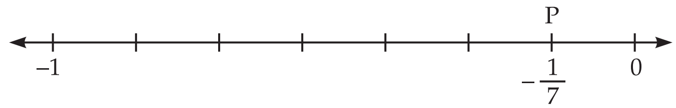
Let us consider a few more examples to recapitulate the representation of rational numbers on the number line.

SOME EXAMPLES

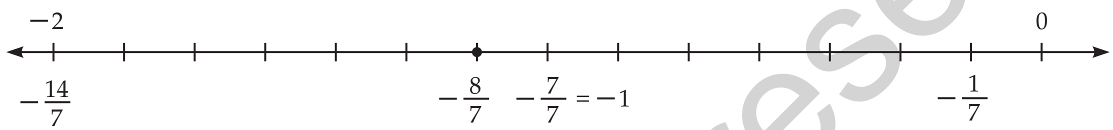
Example 1: Represent $-\frac{1}{7}$ and $-\frac{8}{7}$ on the number line.

Solution: We have to represent $-\frac{1}{7}$ on the number line. Now, $-\frac{1}{7}$ will lie to the left of 0 that is between 0 and –1. As the denominator is 7, we first divide the space between 0 and –1 into 7 equal parts.

Starting from zero, the first of the equally spaced points P will represent $-\frac{1}{7}$; second will be $-\frac{2}{7}$ and so on. The seventh point will be $-\frac{7}{7}$, that is –1.



Similarly, we can represent $-\frac{8}{7}$ on the number line. Now, $-\frac{8}{7}$ will lie between –1 and –2, as it is less than –1 and greater than –2, which can be seen by writing $-\frac{8}{7}$ as the mixed fraction $-1\frac{1}{7}$. We again divide the distance between –1 and –2 into 7 equal parts. The first division to the left of –1 will be $-\frac{8}{7}$.



RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS

As we discussed in the last section, when natural numbers, whole numbers and integers are represented on the number line, it is always possible to count how many numbers lie between any two numbers.

For example, the number of integers between –2 and 2 are 3 (which are –1, 0 and 1). The number of whole numbers between 1 and 5 are 3 (which are 2, 3 and 4).

Now, let us find the rational numbers between two rational numbers.

It is easy to find the rational numbers between $\frac{1}{7}$ and $\frac{6}{7}$, as their denominators are the same. There are four rational numbers, that is, $\frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}$, which lie between $\frac{1}{7}$ and $\frac{6}{7}$.

However, we can also write $\frac{1}{7}$ as $\frac{10}{70}$ and $\frac{6}{7}$ as $\frac{60}{70}$. Then we can say that there are 49 numbers, $\frac{11}{70}, \frac{12}{70}, \frac{13}{70}, \ldots, \frac{57}{70}, \frac{58}{70}, \frac{59}{70}$ that lie between $\frac{1}{7}$ and $\frac{6}{7}$.

Those can be extended further. Let us see how.

If we write $\frac{1}{7}$ as $\frac{100}{700}$ and $\frac{6}{7}$ as $\frac{600}{700}$, we then obtain 499 rational numbers, that is $\frac{101}{700}, \frac{102}{700}, \ldots, \frac{599}{700}$, between $\frac{1}{7}$ and $\frac{6}{7}$. Similarly, the numbers can go on increasing.

Hence, we conclude that the ***number of rational numbers lying between two rational numbers is not definite.***

SOME EXAMPLES

Example 1: Find any four rational numbers between $-\frac{1}{5}$ and $\frac{1}{5}$.

Solution: The rational numbers $-\frac{1}{5}$ and $\frac{1}{5}$ can be written as $\frac{-10}{50}$ and $\frac{10}{50}$ respectively.

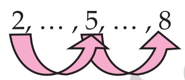
The rational numbers lying between $\frac{-10}{50}$ and $\frac{10}{50}$ are $-\frac{9}{50}, -\frac{8}{50}, -\frac{7}{50}, \ldots 0, \frac{1}{50}, \frac{2}{50}, \ldots, \frac{9}{50}$. Out of these, any four can be picked.

USE OF MEAN TO FIND RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS

Let us explore an alternate way to find a rational number between two rational numbers.

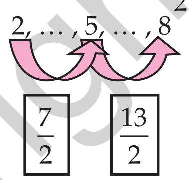
Consider the two rational numbers 2 and 8.

The mean of 2 and 8 is $\frac{2+8}{2} = \frac{10}{2} = 5$. Thus 5 lies between 2 and 8.



Now, to find more rational numbers between 2 and 8, we can find the mean of 2 and 5 and of 5 and 8.

The mean of 2 and 5 is $\frac{2+5}{2} = \frac{7}{2}$, and the mean of 5 and 8 is $\frac{5+8}{2} = \frac{13}{2}$. So, 5, $\frac{7}{2}$ and $\frac{13}{2}$ lie between 2 and 8.



In a similar manner, we can find infinitely many rational numbers between 2 and 8, which can be presented as $2, \ldots, \frac{7}{2}, \ldots, 5, \ldots, \frac{13}{2}, \ldots, 8$.

SOME EXAMPLES

Example 1: Find three rational numbers lying between 3 and 5.

Solution: The mean of 3 and 5 = $\frac{3+5}{2} = \frac{8}{2} = 4$.

We can present it as 3, …, 4, …, 5.

We need 2 more rational numbers.

The mean of 3 and 4 = $\frac{3+4}{2} = \frac{7}{2}$.

The mean of 4 and 5 = $\frac{4+5}{2} = \frac{9}{2}$.

Thus, three rational numbers that lie between 3 and 5 are $\frac{7}{2}$, 4 and $\frac{9}{2}$.

Example 2: Find a rational number between $(x+y)^{-1}$ and $(x^{-1}+y^{-1})$, given that $x = \frac{1}{3}, y = \frac{2}{7}$. (The multiplicative inverse of the number a is denoted by a^{-1}.)

Solution:

$$x = \frac{1}{3}, y = \frac{2}{7}$$

$$x + y = \frac{1}{3} + \frac{2}{7} = \frac{7+6}{21} = \frac{13}{21}$$

$$(x+y)^{-1} = \left(\frac{13}{21}\right)^{-1} = \frac{21}{13} \quad \ldots(i)$$

$$x^{-1} + y^{-1} = \left(\frac{1}{3}\right)^{-1} + \left(\frac{2}{7}\right)^{-1} = \frac{3}{1} + \frac{7}{2} = \frac{6+7}{2} = \frac{13}{2} \quad \ldots(ii)$$

We now need a rational number between $\frac{21}{13}$ and $\frac{13}{2}$.

$$\text{Mean of } \frac{21}{13} \text{ and } \frac{13}{2} = \left(\frac{21}{13} + \frac{13}{2}\right) \div 2$$

$$= \left(\frac{42+169}{26}\right) \div 2 = \left(\frac{211}{26}\right) \div 2 = \frac{211}{26} \times \frac{1}{2} = \frac{211}{52}$$

Hence, the required rational number is $\frac{211}{52}$.

HOTS

(1) Find three rational numbers between $x(y-z)^{-1}$ and $x(y^{-1}-z^{-1})$, given that $x = \frac{2}{5}$, $y = \frac{1}{6}$, $z = \frac{1}{8}$.

(2) Using the mean method, find five rational numbers between –2 and –1.

PRACTICE EXERCISE 1.3

(1) Represent the following rational numbers on the number line:

(i) $\frac{4}{5}$ (ii) $-\frac{9}{11}$ (iii) $\frac{21}{5}$ (iv) $-\frac{13}{4}$

(2) Write any five rational numbers smaller than –2.

(3) Find any five rational numbers between $\frac{1}{7}$ and $\frac{8}{7}$.

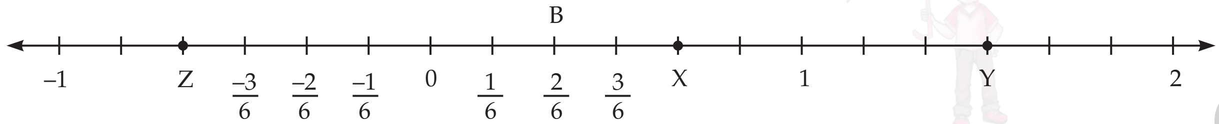
(4) Find any 10 rational numbers between $-\frac{1}{5}$ and $\frac{1}{5}$.

(5) Find five rational numbers between each of the following:

(i) $\frac{2}{7}$ and $\frac{4}{5}$ (ii) $\frac{-3}{4}$ and $\frac{5}{2}$ (iii) $\frac{1}{8}$ and $\frac{1}{2}$

(6) Find three rational numbers between $\frac{2}{5}$ and $\frac{1}{3}$ using the mean method.

(7) Write the rational numbers represented by the points X, Y and Z on the following number line:



WORD PROBLEMS ON RATIONAL NUMBERS

There are many real-life problems that involve rational numbers. These problems can be solved using appropriate operation(s) on rational numbers. Solutions to a few of these word problems are below.

SOME EXAMPLES

Example 1: From a rope 33 m long, as many pieces as possible are cut off, each $3\frac{1}{6}$ m long. Find the number of pieces and the length of the remaining rope.

Solution: Length of rope = 33 m

Length of a piece to be cut from rope = $3\frac{1}{6}$ m

$= \frac{19}{6}$ m

$\therefore$ Number of pieces $= 33 \div \frac{19}{6} = 33 \times \frac{6}{19}$

$= \frac{198}{19} = 10\frac{8}{19}$

Thus, 10 pieces can be cut from the rope.

Length of 10 pieces $= 10 \times \frac{19}{6} = \frac{190}{6}$ m $= \frac{95}{3}$ m.

Length of remaining rope $= \left(33 - \frac{95}{3}\right)$ m $= \frac{33 \times 3 - 95}{3}$ m $= \frac{99 - 95}{3}$ m $= \frac{4}{3}$ m.

Note

Number of pieces can be whole number only.

$\therefore$ $10\frac{8}{19}$ pieces are taken as 10 pieces.

Example 2: If the price of 15 calculators is ₹$2700\frac{3}{5}$ and the price of 6 keychains is ₹$1500\frac{3}{4}$, find the total price of 5 calculators and 8 keychains.

Solution: Price of 15 calculators = ₹$2700\frac{3}{5}$ = ₹$\frac{13503}{5}$

Price of 1 calculator = ₹$\frac{13503}{5} \div 15$

$= \frac{13503}{5} \times \frac{1}{15}$ = ₹$\frac{4501}{25}$

Price of 5 calculators $= 5 \times \frac{4501}{25}$ = ₹$\frac{4501}{5}$

Price of 6 keychains = ₹$1500\frac{3}{4}$ = ₹$\frac{6003}{4}$

Price of 1 keychain = $₹\ \frac{6003}{4} \div 6 = \frac{6003}{4} \times \frac{1}{6} = ₹\ \frac{2001}{8}$

Price of 8 keychains = $₹8 \times \frac{2001}{8} = ₹2001$

Hence, the total price of 5 calculators and 8 keychains

$= ₹\ \frac{4501}{5} + 2001 = ₹\left(\frac{4501 + 10005}{5}\right) = ₹\ \frac{14506}{5}$

$= ₹\ 2901\frac{1}{5}$

Example 3: In a class $\frac{1}{5}$ th of the students in a class are above average; $\frac{2}{3}$ rd are average, and the rest are below average. If the total number of students is 45, how many students are below average?

Solution: Total number of students in the class = 45

Number of students that are above average = $\frac{1}{5} \times 45 = 9$

Number of students that are average = $\frac{2}{3} \times 45 = 30$

Number of students that are below average = 45 – (9 + 30) = 45 – 39 = 6 students

PRACTICE EXERCISE 1.4

(1) The sum of two rational numbers is –7. If one number is $\frac{-21}{5}$, find the other.

(2) If the product of two rational numbers is $-\frac{11}{17}$ and if one of them is $\frac{2}{9}$, then find the other number.

(3) What should be added to $-\frac{19}{5}$ to get $\frac{4}{19}$?

(4) What should be subtracted from $-\frac{3}{5}$ to get $-\frac{4}{9}$?

(5) Find the cost of $3\frac{3}{4}$ metres of cloth at $₹36\frac{2}{3}$ per metre.

(6) The cost of $3\frac{1}{2}$ metres of ribbon is $₹57\frac{3}{4}$. Find the cost of 1 m of the ribbon.

(7) A bus moves at an average speed of $67\frac{4}{5}$ km/hr. How much distance will it cover in $3\frac{1}{2}$ hours?

(8) How many pieces, each of length $3\frac{3}{4}$ m, can be cut from a rope 60 metres long?

(9) The price of 100 one-line notebooks is $₹1500\frac{3}{5}$, whereas the price of 50 five-line notebooks is $₹\ 900\frac{1}{4}$. Suparna wants to give 15 one-line notebooks and 5 five-line notebooks to the school for the underprivileged. Find the total cost of the notebooks.

(10) Rahul purchased 15 blenders from a shopkeeper. The cost of each blender was $₹\ 300\frac{1}{30}$. He returned 6 defective choppers, each costing $₹400\frac{1}{8}$. Who is supposed to pay whom and how much?

PROJECT WORK

Pick up your maths book. Open three spread pages one after the other and make rational numbers by taking the page number of the left page as the numerator and the page number of the right page as the denominator. If the page number in the numerator is even, then take it to be positive, and if page number in the numerator is odd, then take it to be negative. This way you will have three different rational numbers. Verify the closure, commutative, associative and distributive properties for the three rational numbers, and fill the table below. Colour the block green if the property holds well and red if the property does not hold well.

Property	Rational Numbers: $\frac{\square}{\square}, \frac{\square}{\square}$ and $\frac{\square}{\square}$			
	Addition	Subtraction	Multiplication	Division
Closure Law				
Commutative Law				
Associative Law				
Distributive Law				

MULTIPLE CHOICE QUESTIONS

(1) The additive inverse of $\frac{9}{8}$ is:

(a) $\frac{9}{8}$ (b) $-\frac{9}{8}$ (c) $-\frac{8}{9}$ (d) 1

(2) The multiplicative inverse of –7 is:

(a) 7 (b) $\frac{1}{7}$ (c) $-\frac{1}{7}$ (d) 1

(3) Given $a = 11$, $b = 5$, the multiplicative inverse of ab is:

(a) $\frac{1}{11}$ (b) $\frac{1}{16}$ (c) $\frac{1}{6}$ (d) $\frac{1}{55}$

(4) The value of $-\frac{9}{0}$ is:

(a) 0 (b) not defined (c) –9 (d) 9

(5) The multiplicative inverse of a rational number is:

(a) –1 (b) 0 (c) its reciprocal (d) 1

COMPREHENSIVE EXERCISE

(1) **Using the mean method, find a rational number between $-\frac{2}{9}$ and $\frac{9}{2}$.**

(2) **Himanshu donates $\frac{7}{11}$ of all the money that he has in his bank to an orphanage. If the amount that he donates is ₹77000, what is the total money in his bank before he donates the money?**

(3) **Shanaya and her two friends get a room constructed for ₹72000. Simren contributes $\frac{3}{8}$th of Shanaya's contribution, whereas Samantha contributes $\frac{1}{2}$ of Shanaya's share. How much do the three contribute individually?***

* For more practice questions refer to practice book.

CHAPTER CHECK-UP

- Rational numbers are commutative under addition and multiplication; that is for rational numbers x, y

  (*i*) $x + y = y + x$ (ii) $xy = yx$
- Rational numbers are associative under addition and multiplication; that is for rational numbers x, y, z

  (*i*) $x + (y + z) = (x + y) + z$ (ii) $x.(yz) = (xy).z$
- Zero is the additive identity for rational numbers.
- 1 is the multiplicative identify for rational numbers.
- The additive inverse of the rational number $\frac{p}{q}$ is $\frac{-p}{q}$ and vice versa.
- The multiplicative inverse of the rational numbers $\frac{p}{q}$ is $\frac{q}{p}$, where p and q are both non-zero integers.
- For rational numbers the distributive property of multiplication over addition and subtraction holds well; that is for rational numbers x, y, z

  (i) $x \times (y + z) = (x \times y) + (x \times z)$ (ii) $x \times (y - z) = (x \times y) - (x \times z)$

Chapter 2

Powers and Exponents

INTRODUCTION

Exponents are simply a shorthand notation for multiplying the same number by itself several times. Exponents have a wide application in our day-to-day life. Most often exponents are used when we talk about very large or very small numbers, and exponents make it convenient to read, write and compare such numbers.

A few examples of where exponents are used are as follows:

1. To define a computer's memory; for example, 1 gigabyte of RAM means 10^9 bytes.
2. To measure large distances; for example, the distance from the earth to the moon is 3.8×10^5 km.
3. To count things that grow rapidly; for example, the number of bacteria in a single sneeze is between 10^4 and 10^5.
4. To measure the strength of earthquakes.

RECALL

1. Exponents are shorthand for repeated multiplication of the same thing by itself. For example, the shorthand for multiplying the number 8 four times by itself, $8 \times 8 \times 8 \times 8$, is 8^4.
2. In the example above, the number that is being multiplied by itself, 8, is called the **base**, and 4 is the **exponent** or **index** which stands for the number of times the value is being multiplied.
3. The expression 8^4 is read, eight raised to the power of 4, or 'fourth power of eight'. A number written using exponents is called its **exponential notation** or **index form**.
4. There are two specially named powers: 'to the power 2' is generally called squared, and 'to the power three' is called cubed.
5. For any non-zero integer x, $x^0 = 1$.
6. For any non-zero integer x,
   $(-x)^{\text{odd positive integer}} = -(x^{\text{odd positive integer}})$ and
   $(-x)^{\text{even positive integer}} = +(x^{\text{even positive integer}})$.

EXPONENTIAL NOTATION OF RATIONAL NUMBERS

We know that $2 \times 2 \times 2 = 2^3$ and $(-5) \times (-5) \times (-5) \times (-5) \times (-5) = (-5)^5$.

Now $\left(\frac{3}{5}\right)^4 = \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5} = \frac{3 \times 3 \times 3 \times 3}{5 \times 5 \times 5 \times 5} = \frac{3^4}{5^4}$

Likewise, $\left(\frac{-2}{7}\right)^3 = \frac{-2}{7} \times \frac{-2}{7} \times \frac{-2}{7} = \frac{-2 \times -2 \times -2}{7 \times 7 \times 7} = \frac{(-2)^3}{7^3}$

Thus, for any two non-zero integers x and y and a positive integer n, $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$.

RECIPROCALS

We know that two numbers are called reciprocals or multiplicative inverses of each other if their product is 1.

Thus, for any number x, its reciprocal is $\frac{1}{x}$, as $x \times \frac{1}{x} = 1$.

Therefore, 5 and $\frac{1}{5}$ are reciprocals of each other, and so are $\frac{2}{3}$ and $\frac{3}{2}$.

Likewise, we can say that 8^2 and $\frac{1}{8^2}$ are reciprocals of each other.

Note

To find the reciprocal of a non-zero rational number, we interchange the numerator and the denominator.

In the previous section, we learnt that if x and y are any two non-zero integers and n is any positive integer, then $\frac{x^n}{y^n} = \left(\frac{x}{y}\right)^n$.

Therefore, the reciprocal of $\left(\frac{x}{y}\right)^n$ is the same as the reciprocal of $\frac{x^n}{y^n}$, which is $\frac{y^n}{x^n}$ or $\left(\frac{y}{x}\right)^n$. Thus, to find the reciprocal of a rational number, with a positive exponent, find the reciprocal of the rational number and then take its positive exponent.

Note

For any non-zero rational number $\left(\frac{x}{y}\right)$, $\left(\frac{x}{y}\right)^0 = \frac{x^0}{y^0} = \frac{1}{1} = 1.$

POWERS WITH NEGATIVE EXPONENTS

We know that $8^0 = 1$.

Starting from 0, if we increase the exponent we get a positive exponent which indicates the number of times the base has to be used in a multiplication. For example, $8^2 = 8 \times 8$. So, what would a negative exponent indicate? Negative is the opposite of positive, and the opposite of multiplication is division. So, starting from 0, if we decrease the exponent we get a negative exponent which indicates the number of times the base has to be used in a division.

Therefore, $8^{-2} = (1 \div 8) \div 8$.

Moreover, 8^{-2} can also be written as $\frac{1}{8 \times 8}$ or $\frac{1}{8^2}$.

To strengthen the concept further, let us observe the pattern below:

$$8^2 = 8 \times 8 = 64$$
$$8^1 = 8 = \frac{64}{8} = \frac{8^2}{8}$$
$$8^0 = 1 = \frac{8}{8} = \frac{8^1}{8}$$

Note

For any three integers a, b and c,

$(a \div b) \div c = \frac{a/b}{c} = \frac{a/b}{c/1} = \frac{a}{b} \times \frac{1}{c} = \frac{a}{b \times c}$.

As the exponent decreases by 1, the value becomes one-eighth of the previous value that is, the value becomes the previous value divided by the base 8. Continuing in the same manner,

$$8^{-1} = \frac{8^0}{8} = \frac{1}{8}$$

$$8^{-2} = \frac{8^{-1}}{8} = \frac{\frac{1}{8}}{8} = \frac{1}{8 \times 8} = \frac{1}{8^2}$$

In general, for any non-zero integer x and a positive integer n, $x^{-n} = \frac{1}{x^n}$.

Thus, to find the negative exponent of an integer (x^{-n}), we first find the positive exponent $\left(x^n\right)$ of the integer and then take its reciprocal $\left(\frac{1}{x^n}\right)$.

Note

(1) For any non-zero integer x and a positive integer n, x^{-n} and x^n are the reciprocals or multiplicative inverses of each other.

(2) For any non-zero integer x, $x^{-1} = \frac{1}{x}$. Thus, x^{-1} is the reciprocal or multiplicative inverse of x.

RATIONAL NUMBER WITH NEGATIVE EXPONENT

For any two non-zero integers x and y and a positive integer n,

$$\left(\frac{x}{y}\right)^{-n} = \frac{1}{\left(\frac{x}{y}\right)^n} = \frac{1}{\frac{x^n}{y^n}} = \frac{1 \times y^n}{x^n} = \frac{y^n}{x^n} = \left(\frac{y}{x}\right)^n$$

For any three non-zero integers a, b and c,

$$a \div (b \div c) = \frac{a}{b/c} = \frac{a/1}{b/c} = \frac{a}{1} \times \frac{c}{b} = \frac{a \times c}{b}.$$

Thus, to find negative exponent of a rational number $\left(\frac{x}{y}\right)^{-n}$, we first find the reciprocal of the rational number, that is, $\left(\frac{y}{x}\right)$ and then take its positive exponent $\left(\frac{y}{x}\right)^n$.

Note

For any integer n, $1^n = 1$.

Also, for any non-zero integer x and any positive integer n, $\frac{1}{x^{-n}} = \left(\frac{1}{x}\right)^{-n} = x^n$.

EXPANDED FORM USING EXPONENTS

In the last grade, we learnt how to write the expanded form of numbers such as 43712 using exponents:

$$\begin{aligned} 43712 &= 4 \times 10000 + 3 \times 1000 + 7 \times 100 + 1 \times 10 + 2 \times 1 \\ &= 4 \times 10^4 + 3 \times 10^3 + 7 \times 10^2 + 1 \times 10^1 + 2 \times 10^0 \end{aligned}$$

We use negative exponents for writing the expanded form of decimal numbers such as 5639.812:

$$\begin{aligned} 5639.812 &= 5 \times 1000 + 6 \times 100 + 3 \times 10 + 9 \times 1 + 8 \times \frac{1}{10} + 1 \times \frac{1}{100} + 2 \times \frac{1}{1000} \\ &= 5 \times 10^3 + 6 \times 10^2 + 3 \times 10^1 + 9 \times 10^0 + 8 \times 10^{-1} + 1 \times 10^{-2} + 2 \times 10^{-3} \end{aligned}$$

SOME EXAMPLES

Example 1: Write $\frac{81}{625}$ in the exponential form.

Solution: $\frac{81}{625} = \frac{3 \times 3 \times 3 \times 3}{5 \times 5 \times 5 \times 5} = \frac{3^4}{5^4} = \left(\frac{3}{5}\right)^4$

Example 2: Find the value of x if $4^x = 256$.

Solution: We have $4^x = 256 = 4 \times 4 \times 4 \times 4 = 4^4$

$\Rightarrow x = 4$

> If $x^m = x^n$, then $m = n$ only if the base x is different from 1 or –1.
> For example, $1^5 = 1^4 = 1$.
> Here the bases are the same, but the powers are different.

Example 3: Evaluate $(7^0 + 3^0)^6$.

Solution: $(7^0 + 3^0)^6 = (1 + 1)^6 = 2^6 = 64$

Example 4: Evaluate $\left(\frac{-2}{5}\right)^{-3}$

Solution: $\left(\frac{-2}{5}\right)^{-3} = \left(\frac{5}{-2}\right)^3 = \frac{5^3}{(-2)^3} = \frac{5^3}{-(2^3)} = \frac{125}{-8} = -\frac{125}{8}$

Example 5: Find the reciprocal of each of the following:

(i) $\left(\frac{-4}{5}\right)^2$ (ii) $\left(\frac{1}{3}\right)^{-5}$

Solution: (i) Reciprocal of $\left(\frac{-4}{5}\right)^2 = \left(\frac{5}{-4}\right)^2 = \frac{5^2}{(-4)^2} = \frac{5^2}{4^2} = \frac{25}{16}$

(ii) Reciprocal of $\left(\frac{1}{3}\right)^{-5} = \left(\frac{1}{3}\right)^5 = \frac{1^5}{3^5} = \frac{1}{243}$

PRACTICE EXERCISE 2.1

(1) Find the value of each of the following:

(i) $(-3)^2$ (ii) $(4)^5$ (iii) $(-7)^3$ (iv) $(-1)^{16}$

(2) Write each of the following in the exponent form:

(i) $(-2) \times (-2) \times (-2) \times (-2) \times (-2)$ (ii) $p \times p \times q \times q \times q$ (iii) $\frac{3}{7} \times \frac{3}{7} \times \frac{3}{7}$

(3) Find the value of x if $5^x = \frac{1}{625}$.

(4) Find the values of the following:

(i) $1^0 + 2^0 + 3^0 + 4^0 + 5^0$ (ii) $\left(7^0\right)^5$ (iii) $\left(2^3\right)^0 + \left(2^0\right)^3$ (iv) 9×9^0

(5) Write each of the following with a positive exponent:

(i) 5^{-32} (ii) $\left(\frac{2}{5}\right)^{-7}$ (iii) $\left(-\frac{3}{8}\right)^{-12}$

(6) Evaluate each of the following:

(i) 3^{-2} (ii) $\left(\frac{-3}{4}\right)^{3}$ (iii) $\left(\frac{2}{7}\right)^{-3}$

(7) Write the reciprocal of each of the following:

(i) 16^{-7} (ii) $\left(\frac{2}{4}\right)^{-4}$ (iii) $\left(\frac{-3}{4}\right)^{3}$ (iv) $\left(\frac{-2}{-7}\right)^{11}$

(8) Write each of the following in the expanded form using exponents:

(i) 2357.328 (ii) 432.476 (iii) 38.0325 (iv) 926.485

LAWS OF EXPONENTS

In the last grade, we studied various laws of exponents for non-zero integral base and positive exponents.

RECALL

For any two non-zero integers x, y and positive integers m and n, we have the following:

(i) $x^m \times x^n = x^{m+n}$ (ii) $\frac{x^m}{x^n} = x^{m-n}, m > n$ (iii) $(x^m)^n = x^{mn}$

(iv) $(xy)^n = x^n y^n$ (v) $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ (vi) $x^0 = 1$

In this section, we shall extend these laws to negative exponents. Let us restate the laws of exponents and verify them for negative exponents.

PRODUCT RULE – MULTIPLYING POWERS WITH THE SAME BASE

For a non-zero integer x and any two integers m and n, we have $x^m \times x^n = x^{m+n}$

Example: $2^{(-3)} \times 2^{(-5)} = \frac{1}{2^3} \times \frac{1}{2^5} = \frac{1}{2^3 \times 2^5} = \frac{1}{2^{3+5}} = \frac{1}{2^8} = 2^{-8}$ and $2^{(-3)+(-5)} = 2^{-8}$

Thus, $2^{-3} \times 2^{-5} = 2^{(-3)+(-5)}$ is verified.

QUOTIENT RULE – DIVIDING POWERS WITH THE SAME BASE

For a non-zero integer x and any two integers m and n, we have $\frac{x^m}{x^n} = x^{m-n}$

Example: $\frac{5^{-3}}{5^{-2}} = 5^{-3} \times \frac{1}{5^{-2}} = \frac{1}{5^3} \times 5^2 = \frac{5^2}{5^3} = 5^{2-3} = 5^{-1}$ and $5^{(-3)-(-2)} = 5^{-3+2} = 5^{-1}$

Thus, $\frac{5^{-3}}{5^{-2}} = 5^{(-3)-(-2)}$ is verified.

TAKING THE POWER OF A POWER

For a non-zero integer x and any two integers m and n, we have $\left(x^m\right)^n = x^{mn}$

Example: $\left(4^{-3}\right)^{-2} = \left(\frac{1}{4^3}\right)^{-2} = \frac{1}{\left(\frac{1}{4^3}\right)^2} = \frac{1}{\frac{1}{\left(4^3\right)^2}} = \frac{1}{\frac{1}{4^{3\times 2}}} = 4^{3\times 2} = 4^6$ and $4^{(-3)\times(-2)} = 4^6$

Thus, $\left(4^{-3}\right)^{-2} = 4^{(-3)\times(-2)}$ is verified.

MULTIPLYING POWERS WITH THE SAME EXPONENT

For any two non-zero integers x and y and any integer m, we have $x^m \times y^m = (xy)^m$

Example: $3^{-5} \times 7^{-5} = \frac{1}{3^5} \times \frac{1}{7^5} = \frac{1}{3^5 \times 7^5} = \frac{1}{(3\times 7)^5} = \frac{1}{(21)^5} = (21)^{-5} = (3\times 7)^{-5}$

Thus, $3^{-5} \times 7^{-5} = (3\times 7)^{-5}$ is verified.

DIVIDING POWERS WITH THE SAME EXPONENT

For any two non-zero integers x and y and any integer m, we have $\frac{x^m}{y^m} = \left(\frac{x}{y}\right)^m$

Example: $\frac{3^{-4}}{5^{-4}} = \frac{\frac{1}{3^4}}{\frac{1}{5^4}} = \frac{1}{3^4} \times \frac{5^4}{1} = \frac{5^4}{3^4} = \left(\frac{5}{3}\right)^4$ and $\left(\frac{3}{5}\right)^{-4} = \left(\frac{5}{3}\right)^4$

Thus, $\frac{3^{-4}}{5^{-4}} = \left(\frac{3}{5}\right)^{-4}$ is verified.

ZERO EXPONENTS

For any non-zero integer x, we have $x^0 = 1$

Note

It can easily be verified that the above laws hold true if we take the base to be a rational number. That is, if $x = \frac{a}{b}$ and $y = \frac{p}{q}$, where a, b, p and q are non-zero integers and m and n are any two integers, then

(i) $\left(\frac{a}{b}\right)^m \times \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^{m+n}$

(ii) $\frac{\left(\frac{a}{b}\right)^m}{\left(\frac{a}{b}\right)^n} = \left(\frac{a}{b}\right)^{m-n}$

(iii) $\left(\left(\frac{a}{b}\right)^m\right)^n = \left(\frac{a}{b}\right)^{mn}$

(iv) $\left(\frac{a}{b}\right)^n \times \left(\frac{p}{q}\right)^n = \left(\frac{a}{b} \times \frac{p}{q}\right)^n$

(v) $\frac{\left(\frac{a}{b}\right)^n}{\left(\frac{p}{q}\right)^n} = \left[\frac{\frac{a}{b}}{\frac{p}{q}}\right]^n$

(vi) $\left(\frac{a}{b}\right)^0 = 1$

SOME EXAMPLES

Example 1: Express

(i) 125^{-3} as a power with the base 5.

(ii) $\frac{16}{81}$ and $\frac{-16}{81}$ as powers of a rational number.

Solution:

(i) $125^{-3} = \left(5^3\right)^{-3} = 5^{3\times(-3)} = 5^{-9}$ $\quad [(x^m)^n = x^{mn}]$

(ii) $\frac{16}{81} = \frac{2\times2\times2\times2}{3\times3\times3\times3} = \left(\frac{2}{3}\right)^4$ and $\frac{-16}{81} = -\frac{2\times2\times2\times2}{3\times3\times3\times3} = -\left(\frac{2}{3}\right)^4$

Example 2: Simplify and express the result in power notation with a positive exponent.

(i) $\left(\frac{-1}{4}\right)^5 \div \left(\frac{-1}{4}\right)^{-6}$ $\qquad$ (ii) $(2^{-5} \div 2^{-12}) \times 2^{-7}$

Solution:

(i) $\left(\frac{-1}{4}\right)^5 \div \left(\frac{-1}{4}\right)^{-6} = (-4)^{-5} \div (-4)^6 = (-4)^{-5-6} = (-4)^{-11} = \left(\frac{-1}{4}\right)^{11}$ $\quad [x^m \div x^n = x^{m-n}]$

(ii) $(2^{-5} \div 2^{-12}) \times 2^{-9} = \frac{2^{-5}}{2^{-12}} \times 2^{-9} = 2^{-5-(-12)} \times 2^{-9}$ $\quad [x^m \div x^n = x^{m-n}]$

$= (2^{-5+12}) \times 2^{-9} = 2^7 \times 2^{-9} = 2^{7+(-9)} = 2^{-2} = \frac{1}{2^2}$ $\quad [x^m \times x^n = x^{m+n}]$

Example 3: Evaluate the following:

(i) $\left(\frac{3}{5}\right)^{-7} \times \left(\frac{5}{3}\right)^{-4}$ $\qquad$ (ii) $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

Solution:

(i) $\left(\frac{3}{5}\right)^{-7} \times \left(\frac{5}{3}\right)^{-4} = \frac{3^{-7}}{5^{-7}} \times \frac{5^{-4}}{3^{-4}} = 3^{(-7)-(-4)} \times 5^{(-4)-(-7)}$ $\quad [x^m \div x^n = x^{m-n}]$

$= 3^{-7+4} \times 5^{-4+7} = 3^{-3} \times 5^3 = \frac{1}{3^3} \times 5^3 = \frac{5^3}{3^3} = \frac{5\times5\times5}{3\times3\times3} = \frac{125}{27}$

(ii) $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}} = \frac{3^{-5} \times (2\times5)^{-5} \times 5^3}{5^{-7} \times (2\times3)^{-5}}$

$= \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}}$

$= 3^{-5-(-5)} \times 2^{-5-(-5)} \times 5^{-5+3-(-7)}$ $\quad \left[(xy)^m = x^m \times y^m\right]$

$= 3^{-5+5} \times 2^{-5+5} \times 5^{-5+3+7} = 3^0 \times 2^0 \times 5^5 = 1 \times 1 \times 3125 = 3125$

Example 4: Find x so that $(-7)^{2x+3} \times (-7)^9 = (-7)^{24}$.

Solution:

$(-7)^{2x+3} \times (-7)^9 = (-7)^{24}$

$\Rightarrow \quad (-7)^{2x+3+9} = (-7)^{24}$ $\quad [x^m \times x^n = x^{m+n}]$

$\Rightarrow \quad (-7)^{2x+12} = (-7)^{24}$

Because the powers on both sides have the same bases (different from 1 and –1), their exponents must be equal.

$\therefore\ 2x + 12 = 24$

$2x = 24 - 12$

$2x = 12$ or $x = \dfrac{12}{2} = 6$

Example 5: Find the value of the following:

(i) $\left(3^0 + 2^{-1}\right) \div 2^{-2}$ (ii) $\left[\left(\dfrac{1}{2}\right)^{-1} - \left(\dfrac{1}{3}\right)^{-1}\right]^{-2}$ (iii) $\left[6^{-1} + \left(\dfrac{3}{2}\right)^{-1}\right]^{-2}$

Solution:

(i) $\left(3^0 + 2^{-1}\right) \div 2^{-2} = \left(1 + \dfrac{1}{2}\right) \div \dfrac{1}{2^2} = \dfrac{3}{2} \div \dfrac{1}{4} = \dfrac{3}{\cancel{2}} \times \dfrac{\cancel{4}^{2}}{1} = 3 \times 2 = 6$

(ii) $\left[\left(\dfrac{1}{2}\right)^{-1} - \left(\dfrac{1}{3}\right)^{-1}\right]^{-2} = (2-3)^{-2} = (-1)^{-2} = (-1)^2 = 1$

The reciprocal of –1 is –1.

(iii) $\left[6^{-1} + \left(\dfrac{3}{2}\right)^{-1}\right]^{-2} = \left(\dfrac{1}{6} + \dfrac{2}{3}\right)^{-2} = \left(\dfrac{1+4}{6}\right)^{-2} = \left(\dfrac{5}{6}\right)^{-2} = \left(\dfrac{6}{5}\right)^{2} = \dfrac{6 \times 6}{5 \times 5} = \dfrac{36}{25}$

Example 6: Simplify the following:

(i) $\left[\left(\dfrac{2}{13}\right)^{-6} \div \left(\dfrac{2}{13}\right)^{3}\right]^{3} \times \left(\dfrac{2}{13}\right)^{-9}$ (ii) $\left[\left\{\left(\dfrac{-1}{5}\right)^{-2}\right\}^{2}\right]^{-1}$

Solution:

(i) $\left[\left(\dfrac{2}{13}\right)^{-6} \div \left(\dfrac{2}{13}\right)^{3}\right]^{3} \times \left(\dfrac{2}{13}\right)^{-9} = \left[\left(\dfrac{13}{2}\right)^{6} \div \left(\dfrac{2}{13}\right)^{3}\right]^{3} \times \left(\dfrac{13}{2}\right)^{9}$

$= \left[\dfrac{13^6}{2^6} \div \dfrac{2^3}{13^3}\right]^{3} \times \dfrac{13^9}{2^9} = \left[\dfrac{13^6}{2^6} \times \dfrac{13^3}{2^3}\right]^{3} \times \dfrac{13^9}{2^9}$

$= \left[\dfrac{13^{6+3}}{2^{6+3}}\right]^{3} \times \dfrac{13^9}{2^9} = \left[\dfrac{13^9}{2^9}\right]^{3} \times \dfrac{13^9}{2^9} = \dfrac{13^{27}}{2^{27}} \times \dfrac{13^9}{2^9}$

$= \dfrac{13^{27+9}}{2^{27+9}} = \dfrac{13^{36}}{2^{36}}$

(ii) $\left[\left\{\left(\dfrac{-1}{5}\right)^{-2}\right\}^{2}\right]^{-1} = \left\{\left(\dfrac{-1}{5}\right)^{-2}\right\}^{2\times(-1)} = \left\{\left(\dfrac{-1}{5}\right)^{-2}\right\}^{-2} = \left(\dfrac{-1}{5}\right)^{-2\times(-2)} = \left(\dfrac{-1}{5}\right)^{4} = \dfrac{1}{625}$ $[(x^m)^n = x^{mn}]$

Example 7: By what number should $\left(\dfrac{-3}{2}\right)^{-3}$ be divided so that the quotient is $\left(\dfrac{4}{27}\right)^{-2}$?

Solution: Let the required number be x.

According to the question,

$$\frac{\left(\frac{-3}{2}\right)^{-3}}{x} = \left(\frac{4}{27}\right)^{-2} \Rightarrow \frac{\left(\frac{-3}{2}\right)^{-3}}{\left(\frac{4}{27}\right)^{-2}} = x \Rightarrow \frac{\left(\frac{-2}{3}\right)^{3}}{\left(\frac{27}{4}\right)^{2}} = x \Rightarrow x = \frac{\frac{(-2)^3}{3^3}}{\frac{27^2}{4^2}} = \frac{(-2)^3}{3^3} \times \frac{4^2}{27^2} = \frac{(-2)^3}{3^3} \times \frac{(2^2)^2}{(3^3)^2}$$

$$\Rightarrow x = \frac{-2^3}{3^3} \times \frac{2^4}{3^6} = \frac{-2^{3+4}}{3^{3+6}} = \frac{-2^7}{3^9}$$

$$\left[\begin{matrix}(x^m)^n = x^{mn} \\ x^m \times x^n = x^{m+n}\end{matrix}\right]$$

Hence, the required number is $\frac{-2^7}{3^9}$.

Example 8: Fill in the blanks.

144 —($\times 2^{-3}$)→ ( ) —($\times 12^{-1}$)→ ( ) —($\times 3^{-2}$)→ ( )

Solution: $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$

Thus, $144 \times 2^{-3} = 2^4 \times 3^2 \times 2^{-3} = 2^1 \times 3^2 = 18$

Now, $2^1 \times 3^2 \times 12^{-1} = 2^1 \times 3^2 \times (2^2 \times 3)^{-1} = 2^1 \times 3^2 \times 2^{-2} \times 3^{-1} = 2^{-1} \times 3^1 = \frac{3}{2}$ $\left[\begin{matrix}(xy)^m = x^m \times y^m \\ x^m \times x^n = x^{m+n}\end{matrix}\right]$

Also, $2^{-1} \times 3^1 \times 3^{-2} = 2^{-1} \times 3^{-1} = \frac{1}{2 \times 3} = \frac{1}{6}$

144 —($\times 2^{-3}$)→ 18 —($\times 12^{-1}$)→ $\frac{3}{2}$ —($\times 3^{-2}$)→ $\frac{1}{6}$

HOTS

Find the value of x in, $\left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} = (125)^x$.

PRACTICE EXERCISE 2.2

(1) Use the laws of exponents to simplify the following:

(i) $\frac{2^5 \times 2^{-11}}{2^{-7}}$ (ii) $\frac{(a^3)^{-4}}{a^{-5} \times a^{-3}}$ (iii) $(2^4)^{-3} \times (6^3)^{-4}$

(iv) $\frac{4^{-3} \times x^{-5} \times y^{-4}}{4^{-5} \times x^{-8} \times y^3}$ (v) $\frac{(3^{-2})^2 \times (5^2)^{-3} \times (p^{-3})^2}{(3^{-2})^5 \times (5^3)^{-2} \times (p^{-4})^3}$ (vi) $\left(\frac{1}{5}\right)^{45} \times \left(\frac{1}{5}\right)^{-60} - \left(\frac{1}{5}\right)^{28} \times \left(\frac{1}{5}\right)^{-43}$

(2) Write the value of x in each of the following:

(i) $5^x + 5^{x-1} = 750$ (ii) $(-7)^{2x-1} \div (-7)^{-6} = (-7)^3$ (iii) $\left(\frac{5}{7}\right)^{-2} \div \left(\frac{5}{7}\right)^{-14} = \left(\frac{5}{7}\right)^{8x}$

(iv) $\left(-\frac{1}{7}\right)^{-5} \div \left(-\frac{1}{7}\right)^{-7} = (-7)^x$ (v) $\left(\frac{2}{5}\right)^{2x+6} \times \left(\frac{2}{5}\right)^{3} = \left(\frac{2}{5}\right)^{x+2}$ (vi) $\frac{5^x \times 5^3 \times 5^{-2}}{5^{-5}} = 5^{-12}$

(3) Evaluate the following:

(i) $\frac{(-2)^3 \times (-2)^7}{3 \times 4^6}$ (ii) $\left[\left(\frac{1}{7}\right)^{-1} \times \left(\frac{1}{7}\right)^{-2}\right]^{-1}$ (iii) $\frac{10^{-5} \times 9^{-4}}{2^{-4} \times (15)^{-3}}$

(4) Simplify each of the following and hence find their values:

(i) $\left\{\left(\frac{1}{7}\right)^{-1} \times \left(\frac{1}{7}\right)^{-3}\right\} \div \left(\frac{1}{4}\right)^{-2}$ (ii) $\left(\frac{3}{7}\right)^{-5} \times \left(\frac{7}{11}\right)^{-4} \times \left(\frac{11}{3}\right)^{-6}$ (iii) $\left[\left(\frac{3}{4}\right)^{-1} + \left(\frac{6}{5}\right)^{-1}\right]^{-1}$

(iv) $\left[\left(2^{-7} \div 3^{-10}\right) \times 3^{-5}\right]^{0}$ (v) $\left[(-2)^{-2}\right]^{-3}$ (vi) $\left[\left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2}\right] \times 5^{-2}$

(5) Find the value of each of the following:

(i) $(1)^{243}$ (ii) $(-1)^{-243}$ (iii) $(-1)^{300}$

(6) Express $\frac{27}{64}$ and $\frac{-27}{64}$ as powers of a rational number.

(7) By which number should $\left(\frac{1}{3}\right)^{-1}$ be multiplied so that the product is $\left(\frac{-3}{8}\right)^{-1}$?

(8) By which number should $(-6)^{-3}$ be divided so that the quotient is $\left(\frac{3}{4}\right)^{-3}$?

EXPRESSING LARGE AND SMALL NUMBERS IN SCIENTIFIC FORM USING EXPONENTS

Whenever we talk about very large numbers such as the number of cells in a human body (100 billion) or very small numbers such as the size of a red blood cell (0.000007 m), we use exponents. Exponents make it convenient to read, write and compare such numbers.

In the last grade, we learnt how to write very large numbers in the scientific notation or standard form using exponents. We express the large number as a product of a number (lying between 1 and 10, including 1) and a power of 10. For example, the diameter of sun is 1400000000 m. To express this in the standard form, we first write 1400000000 as 1400000000.0 and then shift the decimal nine places to the left (to get number in between 1 and 10) and multiply with 10^9.

So, in the standard form, $1400000000 = 1.4 \times 10^9$.

In a similar manner, we can express very small numbers in the standard form using exponents. Here, we again write the small number as a product of a number (lying between 1 and 10, including 1) and a power of 10. For example, the size of a red blood cell is 0.000007 m. To express this in the standard form we move the decimal six places to the right (to get number between 1 and 10) and divide it by 10^6 (or multiply by 10^{-6}).

So, in the standard form, $0.000007 = 7.0 \times 10^{-6}$.

Note

The exponent in the standard form for very large numbers is positive, whereas it is negative for very small numbers.

SOME EXAMPLES

Example 1: Write the following numbers in the standard form:

(i) 6250000000 (ii) 0.000000714 (iii) 0.00000233

Solution: (i) 6.25×10^9 (ii) 7.14×10^{-7} (iii) 2.33×10^{-6}

Example 2: Express the following numbers in the usual form:

(i) 2.59×10^9 (ii) 6.73×10^{-8} (iii) 2.29×10^{-5}

Solution: (i) 2590000000 (ii) 0.0000000673 (iii) 0.0000229

COMPARING VERY LARGE AND VERY SMALL NUMBERS

We use exponents to write very large or very small numbers in the standard from. This further helps us in comparing such numbers.

Let us compare the radius of the sun (7×10^8 m) with that of the earth (6.4×10^6 m). To do so we take the ratio of the two radii.

$$\frac{\text{Radius of sun}}{\text{Radius of earth}} = \frac{7 \times 10^8 \text{ m}}{6.4 \times 10^6 \text{ m}} = \frac{7 \times 10^{8-6}}{6.4} = \frac{7 \times 10^2}{6.4} = \frac{700}{6.4} = 10^2 \text{ (approximately)} \qquad [x^m \div x^n = x^{m-n}]$$

∴ The radius of the sun is approximately 10^2 times that of the earth.

SOME EXAMPLES

Example 1: In a stack there are 5 books, each of thickness 20 mm, and 5 paper sheets, each of thickness 0.016 mm. What is the total thickness of the stack in metres?

Solution: Thickness of 1 book = 20 mm

Thickness of 5 books = 5×20 mm = 100 mm

Thickness of 1 paper sheet = 0.016 mm

Thickness of 5 paper sheets = $5 \times 0.016 = 0.080$ mm

Total thickness of the stack = 100 + 0.08 = 100.08 mm = 1.0008×10^2 mm = $1.0008 \times 10^2 \times 10^{-3}$ m

= 1.0008×10^{-1} m = 0.10008 m

Example 2: The distance from the earth to the sun is 1.496×10^{11} m, and the distance from the earth to the moon is 3.84467×10^8 m. During solar eclipse the moon comes between the earth and the sun. What is the distance between the moon and the sun at that particular time?

Solution: The distance between the moon and the sun during solar eclipse is obtained by the difference between the distance from the earth to the sun and the distance from the earth to the moon.

The distance between earth and sun is = 1.496×10^{11} m.

The distance between earth and moon is = 3.84467×10^8 m.

The distance between the moon and the sun during solar eclipse

$= 1.496 \times 10^{11} \text{ m} - 3.84467 \times 10^8 \text{ m}$

$= \frac{1496}{1000} \times 10^{11} \text{ m} - 3.84467 \times 10^8 \text{ m}$

> Two numbers in the standard form can be added or subtracted if the numbers have the same exponent.

$= 1496 \times 10^8 \text{ m} - 3.84467 \times 10^8 \text{ m} = 1492.15533 \times 10^8 \text{ m} = 1.49215533 \times 10^3 \times 10^8 \text{ m}$

$= 1.49215533 \times 10^{11}$ m

Example 3: A particular star is at a distance of approximately 8.1×10^{13} km from the earth. Assuming that light travels at 3×10^{8} m per second, find out how long does light takes to travel from the star to reach the earth.

Solution: The distance of the star from the earth = 8.1×10^{13} km = $8.1 \times 1000 \times 10^{13}$ m = 8.1×10^{16} m

Time taken by light to travel a distance of 3×10^{8} m = 1 second

Time taken by light to travel a distance of 8.1×10^{16} m = $\frac{8.1 \times 10^{16}}{3 \times 10^{8}}$ seconds = 2.7×10^{8} seconds

Hence, the time taken by light to travel from the star to reach the earth is 2.7×10^{8} seconds

Example 4: A new born bear weighs 4 kg. How many kilograms would a 5-year-old bear weigh if its weight increases by the power of 2 in 5 years?

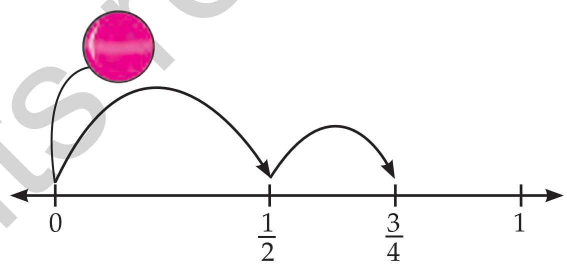
Solution: Weight of a new born bear = 4 kg

The weight increase by the power of 2 in 1 year, so the weight of

1-year-old bear = 4^2, 2-year-old bear = $(4^2)^2 = 4^{2\times2} = 4^4$, 3-year-old bear = $(4^4)^2 = 4^{4\times2} = 4^8$, 4-year-old bear = $(4^8)^2 = 4^{8\times2} = 4^{16}$, 5-year-old bear = $(4^{16})^2 = 4^{16\times2} = 4^{32}$

Thus, a 5-year old bear will weigh 4^{32} kg.

Example 5: A ball is thrown on the 0 point of the number line, bouncing towards 1. It covers half the distance from its current location to 1 with each bounce. So, it will be at $\frac{1}{2}$ after 1 bounce, $\frac{3}{4}$ after 2 bounces and so on.



(i) Make a table listing the ball's location for the first 10 bounces.

(ii) Where will the ball be after n bounces?

(iii) Will the ball ever get to 1? Explain.

Solution: (i)

Number of bounces	Position of the ball on the number line	Number of bounces	Position of the ball on the number line
1	$\frac{1}{2}$	6	$\frac{31}{32} + \frac{1}{64} = \frac{63}{64} = \frac{2^6 - 1}{2^6}$
2	$\frac{1}{2} + \frac{1}{4} = \frac{3}{4} = \frac{2^2 - 1}{2^2}$	7	$\frac{63}{64} + \frac{1}{128} = \frac{127}{128} = \frac{2^7 - 1}{2^7}$
3	$\frac{3}{4} + \frac{1}{8} = \frac{7}{8} = \frac{2^3 - 1}{2^3}$	8	$\frac{127}{128} + \frac{1}{256} = \frac{255}{256} = \frac{2^8 - 1}{2^8}$
4	$\frac{7}{8} + \frac{1}{16} = \frac{15}{16} = \frac{2^4 - 1}{2^4}$	9	$\frac{255}{256} + \frac{1}{512} = \frac{511}{512} = \frac{2^9 - 1}{2^9}$
5	$\frac{15}{16} + \frac{1}{32} = \frac{31}{32} = \frac{2^5 - 1}{2^5}$	10	$\frac{511}{512} + \frac{1}{1024} = \frac{1023}{1024} = \frac{2^{10} - 1}{2^{10}}$

(ii) The ball will be at $\frac{2^n - 1}{2^n}$ after n bounces.

(iii) Suppose that for some value of n, the ball reaches 1. In that case,

$$\frac{2^n - 1}{2^n} = 1$$

$$\Rightarrow \quad 2^n - 1 = 2^n$$

$$\Rightarrow 2^n - 2^n = 1$$

$$\Rightarrow \quad 0 = 1$$, which is absurd.

Thus, the ball will never reach 1.

PRACTICE EXERCISE 2.3

(1) Express the following numbers in the standard form:

(i) 4730000000 (ii) 0.0000000837 (iii) 0.000000000958

(2) Express the following numbers in the usual form:

(i) 4.02×10^{-6} (ii) 8.3×10^{5} (iii) 4×10^{-8} (iv) 5.37×10^{7}

(3) Divide 1673 by 1 trillion, and express the result in the standard form.

(4) The thickness of a wire on computer chip is 0.000003 m, whereas the thickness of a human hair is 0.000006 m. Find the difference between their thicknesses in the standard form.

(5) The thickness of a sheet of paper is 1.5×10^{-3} cm, and the thickness of a human hair is 6×10^{-3} cm. Compare the two.

(6) The number of red blood cells per cubic millimetre of blood is approximately 5.5 million. If the average body contains 5 litres of blood, what is the total number of red blood cells in the body? Express your answer in the standard form. (1 litre = 10,00,000 mm^3)

(7) The cells of a bacterium double every 30 minutes. A scientist begins with a single cell. How many cells will be there after

(i) 12 hours? (ii) 24 hours?

(8) Solar energy, that is energy from the Sun, provides a consistent and steady source of solar power throughout the year. As our non-renewable resources are set to decline in the years to come, it is important for us to move towards renewable sources of energy such as solar, wind, hydropower and biomass energy. India's solar energy is about 5000 trillion kilowatt-hours (kWh) per year. Express this in the standard form.

PROJECT WORK

Prepare a chart of 10 very large numbers, such as the distance between the earth and the sun, the diameter of Earth and the speed of light, and 10 very small numbers, such as the thickness of hair and the charge on electron and present them in the scientific form.

MULTIPLE CHOICE QUESTIONS

(1) $(-1)^{-325}$ is the same as which of the following?

(a) 1 (b) –1 (c) 0 (d) does not exist

(2) $\left(\frac{3}{4}\right)^5 \div \left(\frac{5}{3}\right)^5$ is equal to which of the following?

(a) $\left(\frac{3}{4} \div \frac{5}{3}\right)^5$ (b) $\left(\frac{3}{4} \div \frac{5}{3}\right)^1$ (c) $\left(\frac{3}{4} \div \frac{5}{3}\right)^9$ (d) $\left(\frac{3}{4} \div \frac{5}{3}\right)^{10}$

(3) Which of the following is not the reciprocal of $\left(\frac{2}{3}\right)^4$?

(a) $\left(\frac{3}{2}\right)^4$ (b) $\left(\frac{3}{2}\right)^{-4}$ (c) $\left(\frac{2}{3}\right)^{-4}$ (d) $\frac{3^4}{2^4}$

(4) If $\left(\frac{7}{8}\right)^{2x-3} \times \left(\frac{7}{8}\right)^{-1} = 1$, then x =

(a) $\frac{7}{8}$. (b) 2. (c) 3. (d) None of these.

(5) The usual form for 2.03×10^{-5} is

(a) 0.203. (b) 0.00203. (c) 203000. (d) 0.0000203.

COMPREHENSIVE EXERCISE

(1) Write the reciprocal of each of the following:

(i) 9^{-5} (ii) $\left(\frac{4}{3}\right)^{-5}$ (iii) $\left(\frac{-1}{2}\right)^{-6}$ (iv) $(-7)^{-2} \div 90^{-1}$

(2) If $x = 2$ and $y = -1$, then find the values of the following:

(i) $y^x + x^y$ (ii) $y^x \div x^y$ (iii) $x^2 \times y^2$ (iv) $y^x - x^y$

(3) Planet A is at a distance of 9.35×10^6 km from earth, and planet B is 6.27×10^7 km from earth. Which planet is nearer to earth?*

* For more practice questions refer to practice book.

CHAPTER CHECK-UP

- For any number $a \neq 0$, $a^{-m} = \frac{1}{a^m}$.
- The reciprocal or multiplicative inverse of a^{-m} is a^m.
- For a rational number $\frac{x}{y}$ and an integer m, $\left(\frac{x}{y}\right)^{-m} = \frac{x^{-m}}{y^{-m}}$. In addition, $\left(\frac{x}{y}\right)^{-m} = \left(\frac{y}{x}\right)^{m}$.
- The reciprocal of $\left(\frac{x}{y}\right)^{-m}$ is $\left(\frac{y}{x}\right)^{m}$.
- For any two non-zero numbers x and y and any two integers m and n, we have the following laws of exponents:
  - $x^m \times x^n = x^{m+n}$
  - $\frac{x^m}{x^n} = x^{m-n}$, $m > n$
  - $(x^m)^n = x^{mn}$
  - $(xy)^n = x^n y^n$
  - $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$
  - $x^0 = 1$
- Very big and very small numbers can be expressed in the standard form or scientific form , that is, in the form $p \times 10^n$, where p lies between 1 and 10, including 1 but excluding 10, and n is an integer.

WEBLINKS:

http://www.mathworksheets4kids.com/exponents.html

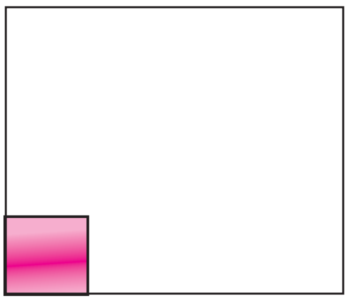
http://www.math-play.com/Exponents-Jeopardy/Exponents-Jeopardy.html

Chapter 3

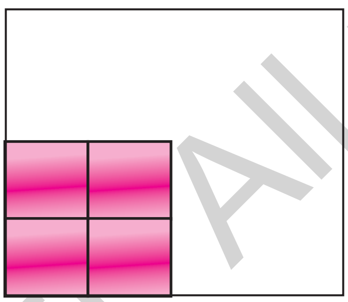
Squares and Square Roots

INTRODUCTION

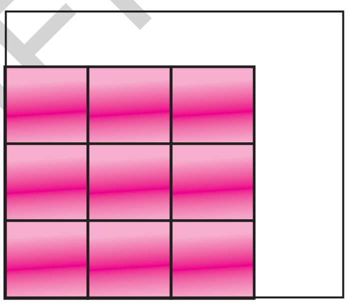
In this chapter, we are going to discuss about squares and square roots. Consider a square floor. Suppose we want to place tiles on this floor in such a manner that the number of tiles in both horizontal and vertical sides is equal.



Here, the number of tiles is $1 \times 1 = 1$



Here, the number of tiles is $2 \times 2 = 4$



Here, the number of tiles is $3 \times 3 = 9$

If we proceed in this manner, we get the number of tiles as 1, 4, 9, 16 (4×4), 25 (5×5), 36 (6×6), 49 (7×7) and so on. What is common in these numbers? Do you see any pattern?

All these numbers are obtained by multiplying a number by itself. These are called ***square numbers.***

The square of a number is obtained by multiplying the number by itself.

For example, square of 3 = $3 \times 3 = 9$.

For any number n, square of $n = n \times n = n^2$.
The natural numbers which are squares of some natural number are called perfect squares.

Example: $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$, $25 = 5^2$ and so on.

$\therefore$ 1, 4, 9, 16, 25, … are perfect squares.

A number is a perfect square if it can be expressed as the product of pairs of factors. For example a natural number m is a square number (or perfect square) if we can write $m = n^2$ for the natural number n.

Let us check whether 45 is a perfect square or not.

We know 45 lies between 36 and 49, i.e., 6^2 and 7^2. That means, if 45 is a square number, there should be a natural number between 6 and 7 whose square is 45 or we can say there should be a natural number which when multiplied by itself gives 45.

But there is no natural number between 6 and 7; therefore, 45 is not a square of any natural number or we can say *45 is not a perfect square*.

Observe the table below; it gives the squares of the first 100 natural numbers.

n	n^2	n	n^2	n	n^2	n	n^2
1	1	26	676	51	2601	76	5776
2	4	27	729	52	2704	77	5929
3	9	28	784	53	2809	78	6084
4	16	29	841	54	2916	79	6241
5	25	30	900	55	3025	80	6400
6	36	31	961	56	3136	81	6561
7	49	32	1024	57	3249	82	6724
8	64	33	1089	58	3364	83	6889
9	81	34	1156	59	3481	84	7056
10	100	35	1225	60	3600	85	7225
11	121	36	1296	61	3721	86	7396
12	144	37	1369	62	3844	87	7569
13	169	38	1444	63	3969	88	7744
14	196	39	1521	64	4096	89	7921
15	225	40	1600	65	4225	90	8100
16	256	41	1681	66	4356	91	8281
17	289	42	1764	67	4489	92	8464
18	324	43	1849	68	4624	93	8649
19	361	44	1936	69	4761	94	8836
20	400	45	2025	70	4900	95	9025
21	441	46	2116	71	5041	96	9216
22	484	47	2209	72	5184	97	9409
23	529	48	2304	73	5329	98	9604
24	576	49	2401	74	5476	99	9801
25	625	50	2500	75	5625	100	10000

PROPERTIES OF SQUARE NUMBERS

PROPERTY 1

If we study the table of squares of first 100 natural numbers discussed in the previous section, we observe that every square number (i.e. value of n^2) ends with any one of the digits 0, 1, 4, 5, 6 or 9; that is, the units place of the square numbers is 0, 1, 4, 5, 6 or 9.

In other words, a square number cannot have its unit's digit as 2, 3, 7 or 8.

Note

A square number cannot end with digits 2, 3, 7 or 8, but it does not mean that every number ending with digit 0, 1, 4, 5, 6 or 9 will be a square number. For example, 131 is not a square number.

PROPERTY 2

On studying the table of squares of the first 100 natural numbers, if we pick up a few square numbers whose units place is 1, we observe that

$$1^2 = 1$$

$$9^2 = 81$$

$$11^2 = 121$$

$$19^2 = 361$$

$$21^2 = 441$$

$$29^2 = 841$$

The square numbers whose units place is 1 are squares of the numbers whose units digit is either 1 or 9. That is, *if a number has 1 or 9 in the units place, then its square has 1 in the units place.*

PROPERTY 3

From the table of squares, picking up a few square numbers whose units digit is 9, we observe that

$3^2 = 9$, $7^2 = 49$, $13^2 = 169$ and $17^2 = 289$,

i.e. *if a number has 3 or 7 in the units place, then its square has 9 in its units place.*

PROPERTY 4

On studying the table of squares of natural numbers and picking up the square numbers whose units digit is 6, we observe that

$4^2 = 16$, $6^2 = 36$, $14^2 = 196$ and $16^2 = 256$,

i.e. *if a number has 4 or 6 in the unit place, then its square has 6 in the units place.*

PROPERTY 5

Look at the squares of 10, 20, 30 …100, 200, …

$10^2 = 100$	$20^2 = 400$	$100^2 = 10000$	$200^2 = 40000$
One Zero — Two Zeros	One Zero — Two Zeros	Two Zeros — Four Zeros	Two Zeros — Four Zeros

What do you observe here?

The number 10 has one zero at the end, and its square: that is 100 has two zeros at the end. Again, 20 has one zero at the end and the square has two zeros at its end, and 200 has two zeros at the end and its square 40000 has four zeros at the end. So, we can say that

if a number has m zeros at the end then its square has 2m zeros at the end.

In other words, a *square number can have only even number of zeros at the end.*

Note

(1) A number ending with odd number of zeros can never be a perfect square.

(2) A number ending with even number of zeros need not be a perfect square. For example, 1200 has two zeros at the end but is not a square number.

PROPERTY 6

Square of an even number is always even and square of an odd number is always odd.

Let us look at the examples.

$2^2 = 4$: Even → Even

$5^2 = 25$: Odd → Odd

$3^2 = 9$: Odd → Odd

$8^2 = 64$: Even → Even

PROPERTY 7

If a number ends with digit 5, its square also ends with digit 5.

Let us look at some of the examples where the units digit of the square number is 5.

$15^2 = 225$; $35^2 = 1225$; $95^2 = 9025$

PROPERTY 8

Perfect squares are always positive.

Example:

$(-3)^2 = 9$; $3^2 = 9$; $(-12)^2 = 144$; $12^2 = 144$

HOTS

Is the sum of the numbers 33333333 and 55555555 a perfect square?

SOME EXAMPLES

Example 1: Out of the following numbers, identify the numbers that are not perfect squares. 625, 4000, 144, 537, 196

Solution: A perfect square cannot end with digits 2, 3, 7 and 8, therefore 537 cannot be a perfect square.

Also, 4000 is not a perfect square as there are three zeros, that is, odd number of zeros at the end of 4000.

Example 2: Is it possible that $229 \times 229 = 52440$?

Solution: No, the square of 229 can never be 52440, as 229^2 should end with digit 1.

Example 3: Which of the squares 13^2, 27^2, 132^2, 64^2, 279^2 would end with digit 6?

Solution: We know that square of a number ends with digit 6 only if the number ends in digit 4 or 6. Therefore, only 64^2 will end with digit 6.

Example 4: What will be the units digit in the square of the following numbers?

(i) 6353 (ii) 37600 (iii) 47 (iv) 55

Solution:

(i) As units digit of the number 6353 is 3, the units digit in of the square of 6353, that is, $(6353)^2$, will be 9.

(ii) As units digit of the number 37600 is 0, the units digit of the square of 37600, that is $(37600)^2$, will be 0.

(iii) As units digit of the number 47 is 7, the units digit of the square of 47, that is, $(47)^2$, will be 9.

(iv) As units digit of the number 55 is 5, the units digit of the square of 55, that is, $(55)^2$, will be 5.

Example 5: What will be the number of zeros at the end of square of 17,000?

Solution: As 17000 has three zeros at its end, its square will have 6 zeros at its end.

Example 6: State whether the squares of the following natural numbers will be odd or even:

(i) 42 (ii) 397 (iii) 543 (iv) 28 (v) 595

Solution: (i) Even (ii) Odd (iii) Odd (iv) Even (v) Odd

PATTERNS IN SQUARE NUMBERS

(I) ***Observe the pattern formed if we add odd numbers in order.***

1 [first odd number] $= 1 = 1^2$

$1 + 3$ [first two odd numbers] $= 4 = 2^2$

$1 + 3 + 5$ [first three odd numbers] $= 9 = 3^2$

$1 + 3 + 5 + 7$ [first four odd numbers] $= 16 = 4^2$

$1 + 3 + 5 + 7 + 9$ [first five odd numbers] $= 25 = 5^2$

$1 + 3 + 5 + 7 + 9 + 11$ [first six odd numbers] $= 36 = 6^2$

and so on.

Here, we can observe that $1 + 3 + 5 + 7 + 9 + 11 \ldots + 2n–1 ;= n^2$ that is, ***the square of any natural number n equals the sum of first n odd numbers.***

(II) ***The square of any odd number can be expressed as the sum of two consecutive positive numbers.***

$3^2 = 9 = 4 + 5$

First Number: $\frac{3^2 - 1}{2}$ Second Number: $\frac{3^2 + 1}{2}$

Similarly, $5^2 = 25 = 12 + 13$

First Number: $\frac{5^2 - 1}{2}$ Second Number: $\frac{5^2 + 1}{2}$

And $7^2 = 49 = 24 + 25$; $9^2 = 81 = 40 + 41$

Note

For any odd natural number n, we can write its square as $\frac{n^2 - 1}{2} + \frac{n^2 + 1}{2}$.

Note

Here, the converse need not be true, i.e. it is not necessary that the sum of any two consecutive natural numbers is a perfect square. For example, 3 + 4 = 7 is not a perfect square.

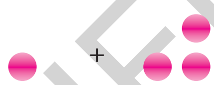
(III) ***Sum of any two consecutive triangular numbers is a square number.***

1 3 6 10 15

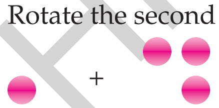
Here, we can observe that 1, 3, 6, 10, 15, 21, … are triangular numbers. If we add any two of the consecutive triangular numbers, we get a square number.

What happens if we combine two consecutive triangles representing the triangular numbers above?

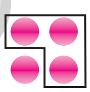
1 + 3



Rotate the second triangle through an angle of 90° anticlockwise.

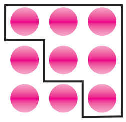
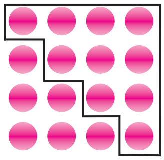


Now, combine the two.



$1 + 3 = 4 = 2^2$

Geometrically, when two triangles, representing triangular numbers are combined, a square is formed. From this we can derive the property that when two consecutive triangular numbers are added, it results in a square number.

$3 + 6 = 9 = 3^2$ and $6 + 10 = 16 = 4^2$

So,

$1 + 3 = 4 = 2^2$,

$3 + 6 = 9 = 3^2$,

$6 + 10 = 16 = 4^2$,

$10 + 15 = 25 = 5^2$,

$15 + 21 = 36 = 6^2$ and so on.

(IV) ***Between the square of n and the square of (n + 1), there are 2n non-square numbers.***

Let us start by checking for 1 and 2. Here, $n = 1$ and $n + 1 = 2$.

So, $n^2 = 1^2 = 1$ and

$(n + 1)^2 = 2^2 = 4$.

The non-square numbers between 1 and 4 are 2 and 3.

So, we have 1^2, <u>2, 3</u>, 2^2.

> For $n = 1$,
> There are 2 (= 2 × 1) non-square numbers between 1^2 and 2^2.

Similarly, if we take $n = 2$,

$n^2 = 2^2 = 4$ and $(n + 1)^2 = 3^2 = 9$.

The non-square numbers that lie between 2^2 and 3^2 are 5, 6, 7, 8.

This can be written as 2^2, <u>5, 6, 7, 8,</u> 3^2.

> There are 4 (= 2 × 2) non-square numbers between 2^2 and 3^2.

Proceeding in the same manner, between 5^2 and 6^2 (i.e., between 25 and 36) the non-square numbers are 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, which are 10 ($= 2 \times 5 = 2 \times n$, where $n = 5$) in number.

Between 6^2 and 7^2, there will be $2 \times n = 2 \times 6 = 12$, since $n = 6$, non-square numbers, which are 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48.

(V) ***Product of two consecutive even or odd natural numbers is equal to 1 less than the square of that natural number which comes in between.***

Let us take an example. Consider two consecutive even numbers 12 and 14. The only natural number that lies between 12 and 14 is 13.

So, $n = 13$.

Let us now check the pattern.

$12 \times 14 = 168$

$n^2 = 13^2 = 169$

So, $n^2 - 1 = 13^2 - 1 = 169 - 1 = 168$

Thus, $12 \times 14 = 168 = 13^2 - 1$

If $n = 13$, 12 can be written as $13 - 1 = n - 1$ and 14 can be written as $13 + 1 = n + 1$

So, we can conclude that,

$$(n-1)\times(n+1)=n^2-1.$$

Let us verify.

$$(n-1)\times(n+1)=n^2-1, \text{ where } n=13$$
$$(13-1)\times(13+1)=12\times14=168=169-1=13^2-1$$

Hence, $(n-1)\times(n+1)=n^2-1$, where $n=13$

Similarly, $42\times44=43^2-1$

This also holds true for two consecutive odd natural numbers.

That is, $9\times11=10^2-1$, $21\times23=22^2-1$.

SOME MORE PATTERNS

(VI) ***A number with n digits has either (2n – 1) or 2n digits in its square.***

For example: One digit number has either one or two digits in its square.

Let us check: $3^2=9$; $4^2=16$ and $9^2=81$.

A two-digit number such as 12 and 19 has either 3 or 4 digits in its square.

Let us check: $12^2=144$, $19^2=361$, $20^2=400$, $95^2=9025$ and $99^2=9801$.

(VII) ***A perfect square leaves the remainder 0 or 1 when divided by 3, i.e. a perfect square can be expressed as 3m or 3m + 1, where m is some natural number.***

Example: $361=3\times120+1$; $289=3\times96+1$; $144=3\times48+0$

Note

A square number is of the form $3m$ or $3m+1$ for some natural number m, but a number of the form $3m$ or $3m+1$ need not be a square number. For example: $13=3\times4+1$ but 13 is not a square number.

(VIII) ***A perfect square leaves the remainder 0 or 1 when divided by 4, i.e. a perfect square can be expressed as 4m or 4m + 1, for some natural number m.***

Example: $361=4\times90+1$

$289=4\times72+1$

$144=4\times36+0$

Note

A square number is of the form $4m$ or $4m+1$ for some natural number m, but a number of the form $4m$ or $4m+1$ need not to be a square number. For example, $440=4\times110$, but 440 is not a square number.

(IX) ***The squares of numbers 1, 11, 111, … form a pattern as shown below:***

$$1^2=1$$
$$11^2=121$$
$$111^2=12321$$
$$1111^2=1234321$$

(X) ***Another pattern we have observed is as follows:***

$$22^2=121\times(1+2+1)$$
$$333^2=12321\times(1+2+3+2+1)$$
$$4444^2=1234321\times(1+2+3+4+3+2+1)$$

Look at the patterns above; here we already know that $121=11^2$ and $1+2+1=2^2$.

So, $121\times(1+2+1)=11^2\times2^2=(11\times2)^2=22^2$.

Similarly, $12321\times(1+2+3+2+1)=111^2\times3^2=(111\times3)^2=333^2$

By simply observing the pattern, we can directly see that

1234321 × (1 + 2 + 3 + 4 + 3 + 2 + 1) = 4444^2 (square of four times 4)

(XI) ***The difference of squares of two consecutive natural numbers is equal to the sum of those natural numbers, i.e.***

$(n + 1)^2 - n^2 = (n + 1) + n$

Example: $5^2 - 4^2 = 5 + 4 = 9$; $25^2 - 24^2 = 25 + 24 = 49$

(XII) Observe the pattern below.

$$25^2 = (2 \times 3)100 + 25 = 625$$
$$35^2 = (3 \times 4)100 + 25 = 1225$$
$$45^2 = (4 \times 5)100 + 25 = 2025$$
$$55^2 = (5 \times 6)100 + 25 = 3025$$
$$65^2 = (6 \times 7)100 + 25 = 4225$$

In the above pattern, we observe ***tens digit is multiplied by its successive number and then multiplied by 100. The result is added to 25.***

HOTS

How many natural numbers lie between 1000^2 and 1001^2?

SOME EXAMPLES

Example 1: Check if the following numbers are a perfect square:

(i) 29 (ii) 81

Solution: (i) We have 29 = (1 + 3 + 5 + 7 + 9) + 4.

∴ 29 cannot be written as the sum of successive odd natural numbers in order starting from 1.

So, 29 is not a perfect square.

We can also check this by subtracting odd natural numbers from 29, one by one, starting from 1:

29 – 1 = 28, 28 – 3 = 25, 25 – 5 =20, 20 – 7 = 13, 13 – 9 = 4. Now we cannot subtract 11 from 4, so we have to stop here.

This means 29 cannot be expressed as the sum of odd natural numbers starting from 1.

∴ 29 is not a square number.

(ii) We have 81 = 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17.

i.e. 81 can be written as the sum of successive odd natural numbers starting from 1.

So, 81 is a perfect square.

We can check this by subtracting odd numbers from 81, one by one, starting from 1. We have 81 – 1 = 80, 80 – 3 = 77, 77 –5 = 72, 72 – 7 = 65, 65 – 9 = 56, 56 – 11 = 45, 45 – 13 = 32, 32 – 15 = 17 and 17 – 17 = 0, which shows 81 can be expressed as the sum of odd natural numbers starting from 1.

∴ 81 is a square number or a perfect square.

Example 2: Find the sum of 9 + 11 + 13 + 15 + 17 + 19.

Solution: We have to find 9 + 11 + 13 + 15 + 17 + 19.

Now, we know that $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = 10^2$.

But we have to find 9 + 11 + 13 + 15 + 17 + 19. For this, we subtract $1 + 3 + 5 + 7 = 4^2$ from 1 + 3 + 5 +7 + 9 + 11 + 13 + 15 + 17 + 19.

So, 9 + 11 + 13 + 15 + 17 + 19

$= (1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19) - (1 + 3 + 5 + 7) = 10^2 - 4^2 = 100 - 16 = 84$.

Example 3: Write the square of 25 as the sum of two consecutive natural numbers.

Solution: $25^2 = 625 = \frac{625-1}{2} + \frac{625+1}{2} = 312 + 313$ [Property 2]

Example 4: How many non-square numbers lie between 25^2 and 26^2?

Solution: Here, $n = 25$; so, there will be $2 \times n = 2 \times 25 = 50$ non-square numbers between 25^2 and 26^2.

Example 5: Can the square of 1543 be 238849?

Solution: Square of 1543 should have either 7 or 8 digits, but 238849 is a six-digit number.

$\therefore$ 238849 cannot be the square of 1543.

Example 6: Evaluate $99^2 - 1$.

Solution: $99^2 - 1 = (99 - 1) \times (99 + 1) = 98 \times 100 = 9800$

Example 7: Use the pattern find 111111^2.

$1^2 = 1$

$11^2 = 121$

$111^2 = 12321$

$1111^2 = 1234321$

$11111^2 = 123454321$

Solution: $111111^2 = 12345654321$

Example 8: Evaluate $999 \times 999 - 998 \times 998$.

Solution: $999^2 - 998^2 = (999 + 998)(999 - 998) = 1997 \times 1 = 1997$

Example 9: Using the pattern as given below, fill in the blank:

$12345654321 \times (1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1) =$ ___________

Solution: $111111^2 \times 6^2 = (111111 \times 6)^2 = (666666)^2$.

Example 10: Use the pattern given below to find 85^2.

$25^2 = (2 \times 3)100 + 25$

$35^2 = (3 \times 4)100 + 25$

$45^2 = (4 \times 5)100 + 25$

$55^2 = (5 \times 6)100 + 25$

Solution: $85^2 = (8 \times 9)100 + 25 = 7200 + 25 = 7225$

PRACTICE EXERCISE 3.1

(1) The numbers below are not the perfect squares. Give reasons for each.

(i) 9367 (ii) 900000 (iii) –289 (iv) 64000 (v) 23498

(2) Without finding the squares, find the units digit in the squares of the following numbers:

(i) 352 (ii) 1328 (iii) 453 (iv) 5676 (v) 599

(vi) 3400 (vii) 67355 (viii) 14237 (ix) 211 (x) 4544

(3) Without actual calculations, find the following sums:

(i) $1 + 3 + 5 + 7 + 9 + 11$

(ii) $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21$

(iii) $11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29$

(iv) $13 + 15 + 17 + 19 + 21 + 23$

(4) Categorise the following numbers as numbers having even or odd squares:

(i) 671 (ii) 1893 (iii) 5544 (iv) 3478 (v) 1007

(5) Express the following square numbers as sum of two consecutive natural numbers:

(i) 441 (ii) 289 (iii) 4225 (iv) 2401 (v) 4489

(6) Draw a square of side length 7 using dots and hence write 7^2 as sum of two triangular numbers.

(7) Find how many non-square natural numbers lie between the following pairs of square numbers.

(i) 51^2 and 52^2 (ii) 10^2 and 11^2 (iii) 17^2 and 18^2 (iv) 21^2 and 22^2

(8) Can square of 591 be 3491? Justify your answer.

(9) Evaluate each of the following without finding the actual square of the numbers:

(i) $79^2 - 1$ (ii) $31^2 - 1$ (iii) $399^2 - 1$

(10) Without finding the squares, evaluate each of the following:

(i) $13^2 - 12^2$ (ii) $221^2 - 220^2$ (iii) $159^2 - 158^2$

(11) Observe each of the patterns below, and fill in the missing numbers/digits.

(i)
$11^2 = 121$
$101^2 = 10201$
$10101^2 = 102030201$
$1010101^2 =$ ________
$\square^2 = 10203040506050403020 1$

(ii)
$8 \times 1 + 1 = 3^2$
$8 \times 3 + 1 = 5^2$
$8 \times 6 + 1 = 7^2$
$8 \times 10 + 1 = 9^2$
$8 \times 15 + 1 = 11^2$
________ $= 13^2$
$8 \times 28 + 1 =$ ________

(iii)
$51^2 = (5^2 + 1) \times 100 + 1^2$
$52^2 = (5^2 + 2) \times 100 + 2^2$
$53^2 = (5^2 + 3) \times 100 + 3^2$
$57^2 =$ ________________
$\square^2 = (5^2 + 9) \times 100 + 9^2$

(iv)
$1^2 + 2^2 + 2^2 = 3^2$
$2^2 + 3^2 + 6^2 = 7^2$
$3^2 + 4^2 + 12^2 = 13^2$
$4^2 +$ __ $+ 20^2 = 21^2$
$6^2 + 7^2 + 42^2 =$ __

(12) Find the sum of first 15 odd natural numbers.

(13) Evaluate $(995)^2$.

(14) A number can be written in the form $3m + 2$, for some natural numbers m. Can this number be a perfect square?

(15) A number can be written in the form $4m + 3$, for some natural number m. Can this number be a perfect square?

FINDING THE SQUARE OF A NUMBER

The square of a number is obtained by multiplying the number by itself. Finding the square of small numbers or one-digit numbers such as 3, 4, 5, 6 and 7,… is very easy and can be done orally. But for two-digits, three-digit numbers, we cannot orally find the square, also the process is time consuming and involves lot of calculations. We shall discuss some alternative methods that helps us find the square of two-digit, three-digit, four-digit numbers and so on quickly and easily. We shall discuss here, three methods that make our calculations to find squares easy.

FIRST METHOD: COLUMN METHOD

We usually use this method to find the squares of two-digit numbers. Suppose x = AB is a two-digit number, where A is tens digit and B is units digit.

Consider the two-digit number 96. Here, A = 9 and B = 6.

Let us follow the steps below:

Step 1: Make a table with three columns and name them I, II and III. Write A^2, 2AB and B^2 as heading of the three columns and find their values corresponding to the number whose square is to be found.

I	II	III
A^2	$2 \times A \times B$	B^2
$9^2 = 81$	$2 \times 9 \times 6 = 108$	$6^2 = 36$

Step 2: Encircle the ones digit of B^2 and add its tens digit, if any, to $2 \times A \times B$.

I	II	III
A^2	$2 \times A \times B$	B^2
81	108 + 3 111	3 (6)

Step 3: Now, encircle the ones digit in $2 \times A \times B$ and add the number formed by the remaining digits, if any, to A^2.

I	II	III
A^2	$2 \times A \times B$	B^2
81 +11 92	108 + 3 11(1)	3 (6)

Step 4: Encircle the number in column I and write the encircled digits at the bottom of each column to obtain the square.

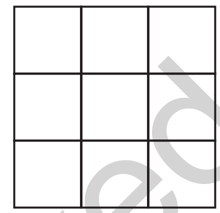
I	II	III
A^2	$2 \times A \times B$	B^2
81 +11 (92)	108 + 3 11(1)	3 (6)
92	1	6

$\therefore \quad 96^2 = 9216$

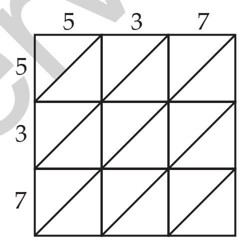
SECOND METHOD: DIAGONAL METHOD

We can use this method to find the squares of number with any number of digits. Let us look at the steps that we need to follow taking an example of a three-digit number, say 537. The steps are as follows:

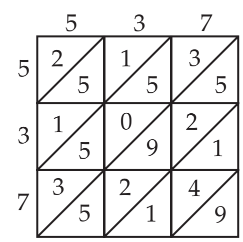
Step 1: To find the square of 'n-digit' number, form an $n \times n$ grid of squares. That is, to find the square of 537, form a 3×3 grid of squares as shown.



Step 2: Draw the diagonals of each of the nine sub-squares and write the digits of number to be squared along the top of the first row and to the left of first column as shown in the adjoining figure.



Step 3: For each sub-square, multiply the digits written on the top with that at the left of it. Write the ones digit of the product below the diagonal and the tens digit of the product above the diagonal.

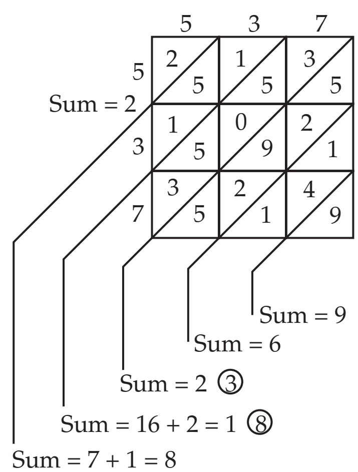


Note

'0' is written above the diagonal if the product is a one-digit number.

Step 4: Now starting from the lowest diagonal, add the digits along the lowest diagonal, write the ones digit of the sum along the diagonal and carry forward the tens digit, if any, to the next diagonal. Starting from the left most digits, the number hence obtained is the square of the given number.

So, here, $537^2 = 288369$



THIRD METHOD: EXPANSION METHOD

This method is useful and convenient to find squares of two digit numbers, though the method can be applied to find square of numbers with any number of digits. This method is similar to the first method, that is, column method.

For example, to find the square of 56, we write 56 as 50 + 6 and then multiply it by itself as shown below:

$$56^2 = (50+6) \times (50+6) = 50(50+6) + 6(50+6) = 50^2 + 50 \times 6 + 6 \times 50 + 6^2 = 2500 + 300 + 300 + 36 = 3136$$

SOME EXAMPLES

Example 1: Find the square of 57 using the column method.

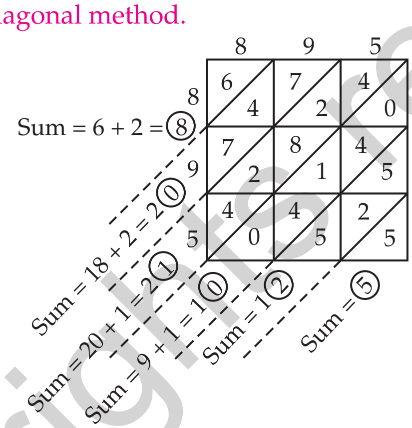
Solution:

I	II	III
A^2	$2 \times A \times B$	B^2
$5^2 = 25$ 25 + 7 32	$2 \times 5 \times 7 = 70$ $70 + 4 = 7④$	7^2 $= 4⑨$
32	4	9

$\therefore \quad 57^2 = 3249$

Example 2: Find square of 895 using the diagonal method.

Solution:



By diagonal method:

$\therefore \quad 895^2 = 801025$

Example 3: Find the square of the following numbers without actual multiplication:

(i) 99 (ii) 37

Solution: (i) $99^2 = (90 + 9) \times (90 + 9)$

$= 90(90 + 9) + 9(90 + 9)$

$= 90^2 + 90 \times 9 + 9 \times 90 + 9^2$

$= 8100 + 810 + 810 + 81$

$= 9801$

(ii) $37^2 = (30 + 7) \times (30 + 7)$

$= 30(30 + 7) + 7(30 + 7)$

$= 30^2 + 30 \times 7 + 7 \times 30 + 7^2$

$= 900 + 210 + 210 + 49$

$= 1369$

SQUARES OF RATIONAL NUMBERS

We know that rational numbers are written in the form of $\frac{p}{q}$, where $q \neq 0$ and p and q are integers. The square of a rational number is obtained by dividing square of its numerator by the square of its denominator, that is, p^2 divided by q^2. That means square of $\frac{p}{q} = \frac{p^2}{q^2}$.

Example: Square of $\frac{4}{5} = \left(\frac{4}{5}\right)^2 = \frac{4^2}{5^2} = \frac{16}{25}$

Square of $\frac{-5}{9} = \left(\frac{-5}{9}\right)^2 = \frac{(-5)^2}{9^2} = \frac{25}{81}$

Square of any number, positive or negative, is always positive.

SOME PATTERNS IN FINDING SQUARES

We have already studied about the squares of the numbers whose units digit is 5. Observe the pattern shown below.

$$25^2 = 2 \times 3 \times 100 + 25 = 625$$

$$105^2 = 10 \times 11 \times 100 + 25 = 11025$$

Thus, we can find the square of a number having its units digit 5 using the following steps:

Step 1: Remove the last digit 5 from the number that is to be squared to obtain a new number. For example, in the number 45, removing 5 we obtain 4.

Step 2: Multiply this number by its successor, i.e., the next consecutive number, and then multiply it by 100. Here, is 4 is multiplied to its successor 5 and then multiplied by 100 i.e. $4 \times 5 \times 100 = 2000$.

Step 3: Add 25 to the product obtained in the last step.

That is, $2000 + 25 = 2025$.

Hence, 2025 is the required square.

SOME EXAMPLES

Example 1: Evaluate: $45^2 + 55^2 + 65^2$.

Solution:

$$45^2 = 4 \times 5 \times 100 + 25 = 2000 + 25 = 2025$$

$$55^2 = 5 \times 6 \times 100 + 25 = 3000 + 25 = 3025$$

$$65^2 = 6 \times 7 \times 100 + 25 = 4200 + 25 = 4225$$

$$\therefore \quad 45^2 + 55^2 + 65^2 = 2025 + 3025 + 4225 = 9275$$

Example 2: Find the squares of the following rational numbers:

(i) $-\frac{2}{3}$ (ii) $\frac{7}{9}$ (iii) $-\frac{2}{7}$ (iv) $\frac{12}{13}$

Solution:

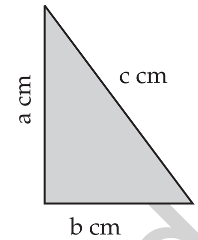
(i) $-\frac{2}{3} = \frac{(-2)^2}{3^2} = \frac{4}{9}$

(ii) $\frac{7}{9} = \frac{7^2}{9^2} = \frac{49}{81}$

(iii) $-\frac{2}{7} = \frac{(-2)^2}{7^2} = \frac{4}{49}$

(iv) $\frac{12}{13} = \frac{12^2}{13^2} = \frac{144}{169}$

PYTHAGOREAN TRIPLET



Consider a right-angled triangle, with sides *a* cm, *b* cm and the hypotenuse is *c* cm.

Then, by Pythagoras property, square of the hypotenuse is equal to the sum of the squares of the other two sides.

That is,

$$c^2 = a^2 + b^2$$

Any three integers satisfying the Pythagoras property are termed as Pythagorean triplets.

That is, for any three integers if square of one number is equal to sum of the squares of the other two numbers then we call them Pythagorean triplets.

For example,

$$3^2 + 4^2 = 5^2$$

Thus, we say that (3, 4, 5) are Pythagorean triplet.

Similarly, $(5^2 + 12^2 = 13^2)$ $(\because 25 + 144 = 169)$

$\therefore$ (5, 12, 13) forms another Pythagorean triplet.

Note

There can be more than one Pythagorean triplet containing one particular number. For example: 5, 12, 13 and (12, 35, 37) both contain 12.

In general, for any natural number $m > 1$, $2m$, $m^2 - 1$ and $m^2 + 1$ form a Pythagorean triplet. Putting $m = 2, 3, 4, 5, \ldots$, we get different sets of Pythagorean triplets.

SOME EXAMPLES

Example 1: Find a Pythagorean triplet one of whose number is 12.

Solution: Let $2m = 12$, then $m = 6$

$\therefore$ $m^2 - 1 = 36 - 1 = 35$ and $m^2 + 1 = 36 + 1 = 37$

$\therefore$ (12, 35, 37) is the required Pythagorean triplet.

Note

In this example, we can't take $m^2 + 1 = 12$ or $m^2 - 1 = 12$ because then we will have $m^2 = 11$ or $m^2 = 13$ which will not give an integral solution.

Example 2: Find a Pythagorean triplet whose smallest member is 7.

Solution: Here, we can use that if *m* is any odd natural number then Pythagorean triplet is $m, \frac{m^2-1}{2}$ and $\frac{m^2+1}{2}$.

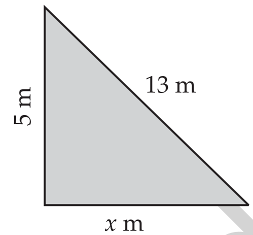
Taking $m = 7$ we get $\frac{m^2-1}{2} = \frac{7^2-1}{2} = 24$ and $\frac{m^2+1}{2} = \frac{7^2+1}{2} = 25$.

Thus, (7, 24, 25) is required Pythagorean tripet.

Note

1. For any odd natural number $m > 1$, Pythagorean triplet is $\left(m, \frac{m^2-1}{2}, \frac{m^2+1}{2}\right)$
2. For any even natural number *m*, except 2, Pythagorean triplet is $\left(m, \frac{m^2-4}{4}, \frac{m^2+4}{4}\right)$

Example 3: A 13 m long ladder is leaned against a wall. The ladder reaches the wall at a height of 5 m. Find the distance between the foot of the ladder and the wall.



Solution: Let the distance between the foot of the ladder and the wall be x.

Height of the wall where the ladder reaches = 5 m

Length of the ladder = 13 m

Since, this forms a right-angled triangle, so (5, x, 13) form a Pythagorean triplet with its smallest side, 5, being an odd natural number.

Thus, the Pythagorean triplet is $\left(m, \frac{m^2 - 1}{2} \text{ and } \frac{m^2 + 1}{2}\right)$

The smallest number is 5, so $m = 5$.

$\therefore \quad m^2 = 25$

$\Rightarrow \quad m^2 + 1 = 26$

$\Rightarrow \quad \frac{m^2 + 1}{2} = \frac{26}{2} = 13$

Now, the third side x is given by $\frac{m^2 - 1}{2} = \frac{25 - 1}{2} = \frac{24}{2} = 12$.

Hence, the distance between the foot of the ladder and the wall = 12 m.

PRACTICE EXERCISE 3.2

(1) Without actual multiplication, find the squares of the following numbers:

(i) 57 (ii) 73 (iii) 81 (iv) 29

(2) Find square of the following natural numbers using column method:

(i) 49 (ii) 38 (iii) 83 (iv) 76

(3) Find the squares of the following natural numbers using diagonal method:

(i) 53 (ii) 97 (iii) 748 (iv) 359

(4) Find the square of the each of the following natural numbers:

(i) 85 (ii) 295 (iii) 505 (iv) 795

(5) Find a Pythagorean triplet whose:

(i) one member is 5.
(ii) one member is 10.
(iii) smallest member is 10.
(iv) smallest member is 14.
(v) whose one member is 15.
(vi) whose greatest member is 101.
(vii) whose one member is 25.
(viii) whose smallest member is 13.

(6) Find the square of the following rational numbers:

(i) $\frac{12}{15}$ (ii) $-\frac{7}{8}$ (iii) $\frac{11}{13}$ (iv) $-\frac{2}{3}$

(7) Verify if the following triplets are Pythagorean triplets or not.

(i) (4, 5, 6) (ii) (6, 8, 10) (iii) (12, 16, 20) (iv) (11, 24, 26)

(8) Evaluate each of the following:

(i) $65^2 + 95^2$ (ii) $995^2 + 495^2$

(9) A 17 m long ladder is leaned against a wall. The ladder reaches the wall at a height of 8 m. Find the distance between the foot of the ladder and the wall.

(10) Rehan walks 12 m south from his house. He then turns towards east and walks 35 m to his teacher's house. While returning back he walks diagonally to reach back his house. What distance did he walk while returning?

SQUARE ROOT

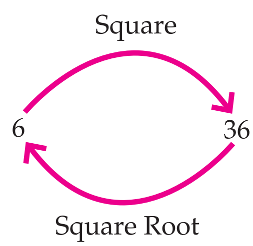
We are very well familiar with the process of finding the square of a number. Now, we shall study the inverse process. We already have studied and calculated squares of various numbers, such as, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$ and $5^2 = 25$.

We read these as 4 is the square of 2, 9 is the square of 3, 16 is the square of 4 and 25 is the square of 5. Now, how do we find a number whose square is 36?

Let us consider an equation $x^2 = 36$.

To solve this equation, we need to find a number whose square is 36. Similar equations can be made to find a number whose square is known. Finding the number, whose square is known, means finding the square root. Finding a square root is the inverse process of finding the square of a number.

Now, it is easy to observe that $x = 6$ satisfies the equation $x^2 = 36$ and so we say that square root of 36 is 6.



Thus, square root of a number can be defined as follows:

> The square root of a number n is that number which when multiplied by itself gives n as the product.

The square root of a number is indicated by the symbol $\sqrt{\ }$ (radical sign), i.e. the square root of n is written as $\sqrt{n}$.

Example:

$$2^2 = 4 \Rightarrow \sqrt{4} = 2; \quad 3^2 = 9 \Rightarrow \sqrt{9} = 3$$

$$4^2 = 16 \Rightarrow \sqrt{16} = 4; \quad 5^2 = 25 \Rightarrow \sqrt{25} = 5$$

Note

We observe $5^2 = 25$ and $(-5)^2 = 25$.

$\therefore$ Square root of 25 can be 5 or –5. In general, the square root of a positive number is either positive or negative.

FINDING SQUARE ROOT OF PERFECT SQUARES

There are different ways of finding the square root of a given perfect square.

FINDING SQUARE ROOT THROUGH REPEATED SUBTRACTION

We have already studied in the patterns for square numbers that every square number can be expressed as the sum of successive odd natural numbers starting from 1.

Example: $49 = 1 + 3 + 5 + 7 + 9 + 11 + 13$

Let us start subtracting successive odd numbers from 49, starting from 1; we will definitely reach '0' after a finite number of steps for perfect square numbers.

Step 1: $49 - 1 = 48$
Step 2: $48 - 3 = 45$
Step 3: $45 - 5 = 40$
Step 4: $40 - 7 = 33$
Step 5: $33 - 9 = 24$
Step 6: $24 - 11 = 13$
Step 7: $13 - 13 = 0$

We have reached a zero after 7 steps.

As we know that the sum of first n odd natural numbers is n^2, the number of steps (subtractions) required to reach '0' gives the value of n.

As 49 is a perfect square such that $n^2 = 49$, so the number of repeated subtractions made to reach zero gives the value of n.

Number of steps = 7

$\therefore \quad \sqrt{49} = 7$

SOME EXAMPLES

Example 1: Find square root of 144 by repeated subtraction.

Solution:

Step 1: $144 - 1 = 143$
Step 2: $143 - 3 = 140$
Step 3: $140 - 5 = 135$
Step 4: $135 - 7 = 128$
Step 5: $128 - 9 = 119$
Step 6: $119 - 11 = 108$
Step 7: $108 - 13 = 95$
Step 8: $95 - 15 = 80$
Step 9: $80 - 17 = 63$
Step 10: $63 - 19 = 44$
Step 11: $44 - 21 = 23$
Step 12: $23 - 23 = 0$

$\therefore$ So, $\sqrt{144} = 12$.

Example 2: Find whether 84 is a perfect square or not.

Solution:

Step 1: $84 - 1 = 83$
Step 2: $83 - 3 = 80$
Step 3: $80 - 5 = 75$
Step 4: $75 - 7 = 68$
Step 5: $68 - 9 = 59$
Step 6: $59 - 11 = 48$
Step 7: $48 - 13 = 35$
Step 8: $35 - 15 = 20$
Step 9: $20 - 17 = 3$
Step 10: $3 - 19 = -16$

As we are not reaching zero by repeated subtraction, so 84 cannot be a perfect square.

Did you notice in the above examples that finding square roots by repeated subtraction method is easy but becomes lengthy, if we have to find the square roots of large numbers. This will involve a large number of steps. So, we can find square roots of a perfect square by other simpler methods discussed in the section ahead.

FINDING SQUARE ROOT THROUGH PRIME FACTORISATION

We have studied many square numbers. Let us perform the prime factorisation of a square number. Consider the square number 225 and find the prime factorisation of 225.

$225 = 3 \times 3 \times 5 \times 5$

3	225
3	75
5	25
5	5
	1

Now, observe here that 3 and 5 both occur twice in the prime factorisation of 225.

Let's find prime factorisation of another square number.

Consider 196.

Here, $196 = 2 \times 2 \times 7 \times 7$.

2	196
2	98
7	49
7	7
	1

Now, observe here that both 2 and 7 occur twice in the prime factorisation of 196.

What is common in both the prime factorisations?

1. Both are prime factorisations of square numbers.
2. Each of the factors of the numbers occur twice in the prime factorisations.

Let us pair the prime factors.

$$225 = 3 \times 3 \times 5 \times 5 = 3^2 \times 5^2 = (3 \times 5)^2 = 15^2$$

So, $\sqrt{225} = 3 \times 5 = 15$

Similarly, $196 = 2 \times 2 \times 7 \times 7 = 2^2 \times 7^2 = (2 \times 7)^2 = 14^2$

So, $\sqrt{196} = 2 \times 7 = 14$

So, to find the square root of a perfect number by prime factorisation, we have to follow the steps below:

Step 1: Find prime factors of the given number by prime factorisation.

Step 2: Arrange these factors and group them in pairs.

Step 3: For every pair, take one factor; that is, if there is a pair such as 2×2, take only one prime factor, i.e. 2.

Step 4: Find the product of the factors obtained in step 3 to get the square root of the given number.

Let us now check if 416 is a square number. For this let us find the prime factors of 416.

2	416
2	208
2	104
2	52
2	26
13	13
	1

$$416 = \overline{2 \times 2} \times \overline{2 \times 2} \times 2 \times 13$$

Since all the factors of 416 are not in pairs, so 416 is not a perfect square; that is, we can say that $\sqrt{416}$ is not a natural number.

SOME EXAMPLES

Example 1: Find the square root of 16384 using prime factorisation.

Solution: We have

$$16384 = \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{2 \times 2}$$

$$\therefore \quad \sqrt{16384} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 128$$

2	16384
2	8192
2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Example 2: Find the smallest number by which 1728 should be multiplied so that the product is a perfect square. Also, find the square root of the product.

Solution: We have,

$$1728 = \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{2 \times 2} \times \overline{3 \times 3} \times 3$$

Clearly, one 3 is left unpaired.

$\therefore$ To get a perfect square, all the prime factors should be in pairs, so, 1728 should be multiplied by 3.

Then the new number becomes

$$1728 \times 3 = 5184$$

So that $\sqrt{5184} = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

$= 2 \times 2 \times 2 \times 3 \times 3 = 72$

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

Example 3: Find the smallest number by which 26325 should be divided so that the number becomes a perfect square. Also, find the square root of this perfect square.

Solution: We have,

$$26325 = \overline{3 \times 3} \times \overline{3 \times 3} \times \overline{5 \times 5} \times 13$$

Clearly, one 13 is left unpaired.

It can be removed by dividing 26325 by 13.

That is $26325 \div 13 = 2025$, which is a perfect square

Also $\sqrt{2025} = 3 \times 3 \times 5 = 45$

3	26325
3	8775
3	2925
3	975
5	325
5	65
13	13
	1

Example 4: Check if 4220 is a perfect square.

Solution: We have,

$4220 = \overline{2 \times 2} \times 5 \times 211$

As, 5 and 211 do not have pairs, so 4220 is not a perfect square.

2	4220
2	2110
5	1055
211	211
	1

Example 5: Find the least square number which is exactly divisible by each one of the numbers 3, 5 and 12.

Solution: The least square number divisible by 3, 5 and 12 should be divisible by their LCM.

LCM (3, 5, 12) = $\underline{2} \times \underline{2} \times 3 \times 5$

Here, we can pair only 2, and 3 and 5 do not appear in pairs. That means the LCM is not a square number.

By multiplying the LCM by 3×5, we can make it a perfect square.

$\therefore$ The least square number divisible by 3, 5, 12 is

$\overline{2 \times 2} \times \overline{3 \times 3} \times \overline{5 \times 5} = 900.$

3	3, 5, 12
2	1, 5, 4
2	1, 5, 2
5	1, 5, 1
	1, 1, 1

Example 6: A total of 1764 plants are to be planted in a garden in such a way that each row should contain as many plants as the number of rows. Find the number of rows and the number of plants in each row.

Solution: Let the number of rows in the garden be x.

$\therefore$ The number of plants in each row = x

$\therefore$ The total number of plants in the garden = $x \times x = x^2$

As per question $x^2 = 1764$

$\Rightarrow \quad x = \sqrt{1764}$

We have $1764 = \overline{2 \times 2} \times \overline{3 \times 3} \times \overline{7 \times 7}$

$\therefore \quad \sqrt{1764} = 2 \times 3 \times 7 = 42$

Hence, there are 42 rows in the garden and each row contains 42 plants.

2	1764
2	882
3	441
3	147
7	49
7	7
	1

Example 7: The students of a class donated ₹1225 for a blind school on Christmas Day. Each student donated as many rupees as the number of students in the class. Find the number of students in the class.

Solution: Let the number of students in the class be x.

So, each student donated ₹x

Total donation from the class = ₹$x \times x$ = ₹x^2

Given that $x^2 = 1225$

$1225 = x^2$

$\Rightarrow \quad \sqrt{1225} = x$

$1225 = \overline{5 \times 5} \times \overline{7 \times 7}$

$\Rightarrow \quad \sqrt{1225} = 5 \times 7 = 35$

Hence, the number of students in the class is 35.

5	1225
5	245
7	49
7	7
	1

Example 8: The product of two numbers is 2475 and their quotient is $\frac{11}{9}$. Find the number.

Solution: Let one number be x. Then the other number will be $\frac{2475}{x}$.

The quotient of the two numbers is given to be $\frac{11}{9}$.

$$\Rightarrow \qquad \frac{x}{\frac{2475}{x}} = \frac{11}{9}$$

$$\Rightarrow \qquad x \div \frac{2475}{x} = \frac{11}{9}$$

$$\Rightarrow \qquad x \times \frac{x}{2475} = \frac{11}{9}$$

$$\Rightarrow \qquad x^2 = \frac{11}{9} \times 2475$$

$$\Rightarrow \qquad x^2 = \frac{27225}{9} = 3025 = \underline{5 \times 5} \times \underline{11 \times 11}$$

$$\Rightarrow \qquad x = 5 \times 11 = 55$$

5	3025
5	605
11	121
11	11
	1

Now, other number is $\frac{2475}{x} = \frac{2475}{55} = 45$.

Hence, the two numbers are 55 and 45.

PRACTICE EXERCISE 3.3

(1) Find the square roots of each of the following numbers using repeated subtraction:
(i) 121 (ii) 100 (iii) 64 (iv) 81

(2) Find the square root of the following numbers by the prime factorization method.
(i) 169 (ii) 225 (iii) 841 (iv) 8281
(v) 9604 (vi) 7744 (vii) 20736

(3) For each of the following numbers, find the smallest whole number by which it should be multiplied so as to get a perfect square number. Also, find the square root of the square number so obtained:
(i) 4802 (ii) 588 (iii) 1536 (iv) 1620 (v) 2028

(4) For each of the following numbers, find the smallest whole number by which it should be divided so as to get a perfect square. Also, find the square root of the square number so obtained.
(i) 29403 (ii) 5120 (iii) 14175 (iv) 1134 (v) 6272

(5) A society collected ₹7744, for the Chennai flood relief, each member contributing as many rupees as there were members. How many members were there in the society?

(6) The area of a square is 5476 m^2. Find the perimeter of the square.

(7) After arranging 680 rose plants in a square, 4 plants are left with the gardener. How many plants are there in each row?

(8) An army general arranges his soldiers for a drill in such a way that the number of rows is the same as the number of columns. In doing so, he finds that 64 soldiers are left out. If the total number of soldiers is 2000, find the number of soldiers in each row.

(9) Find the smallest square number which is exactly divisible by 4, 9 and 10.

(10) Find the smallest square number which is divisible by each of the numbers 6, 9 and 15.

FINDING SQUARE ROOT BY DIVISION METHOD

When we have large numbers or there are a large number of prime factors of a number, then finding square root by prime factorisation method becomes very lengthy and difficult. To overcome this problem, we use long division method.

Before, we start calculating the square root of a number by this method, we should be able to estimate the number of digits of the square root of a square number.

Suppose we want to find the square root of 15376.

First, put bars on the digits of the square number in pairs. If the number of digits is odd, put a bar on a single digit on the extreme left of the number. In the square number 15376, there are five digits, so we put bars as follows:

$\overline{1}\ \overline{53}\ \overline{76}$ — Each pair of digits under a bar is called a period.

Now, count the number of bars. Here, it is 3. So, the number of digits in the square root of 15376 is 3.

Let us also check this by calculating the square root of this number.

To find the square root of a perfect square by long division method, we follow the steps below:

Step 1: Mark periods using bars (as done earlier) on the number given. Here, the number will be written as $\overline{1}\ \overline{53}\ \overline{76}$.

Step 2: Think of a whole number whose square is either equal to or just less than the first period. Take this number as the divisor and as the quotient.

Here it will be 1, as we have 1 in the first period and $1 \times 1 = 1$.

$$\begin{array}{r|l} & 1 \\ \hline 1 & \overline{1}\ \overline{53}\ \overline{76} \end{array}$$

Step 3: Subtract the product of the divisor and the quotient from the first period and bring down the next period to the right of the remainder. This becomes the new dividend.

Since, the difference here is 0, the new dividend is 53.

$$\begin{array}{r|l} & 1 \\ \hline 1 & \overline{1}\ \overline{53}\ \overline{76} \\ & -1 \\ \hline & 0\ 53 \end{array}$$

Step 4: Now new divisor is obtained by adding ones digit of the previous divisor to the previous divisor itself and suffixing a suitable digit to it. So, we write this sum with a blank as the new divisor. The next digit of the quotient is chosen in such a way that the product of new divisor and this chosen digit (of quotient) is equal to or just less than the new dividend.

Let us now check. We have 2_. We have to fill this blank, such that the product of the new divisor with this digit is equal or less than the new dividend, that is, 53.

$$\begin{array}{r|l} & 1 \\ \hline 1 & \overline{1}\ \overline{53}\ \overline{76} \\ & -1 \\ \hline 2\_ & 0\ 53 \end{array}$$

Now, $21 \times 1 = 21$

$22 \times 2 = 44$

$23 \times 3 = 66 > 53$, so we write 2 in the blank space.

Therefore, our new divisor is 22.

Step 5: Repeat step 4 till all the periods have been taken up. Now the quotient so obtained is the square root of the given number. Here it is 124.

$\therefore \quad \sqrt{15376} = 124$

$$\begin{array}{r|r} & 124 \\ \hline 1 & \overline{1}\ \overline{53}\ \overline{76} \\ & -1 \\ \hline 22 & 0\ 53 \\ & -44 \\ \hline 244 & 976 \\ & -976 \\ \hline & 0 \end{array}$$

SOME EXAMPLES

Example 1: Find the square root of (i) 3481 and (ii) 576.

Solution:

(i)

	59
5	$\overline{34}\ \overline{81}$
	−25
109	981
	−981
	0

(ii)

	24
2	$\overline{5}\ \overline{76}$
	−4
44	176
	−176
	0

$\therefore\ \sqrt{3481} = 59$ $\therefore\ \sqrt{576} = 24$

Example 2: Find the least number that when subtracted from 2050 gives a perfect square. Also, find the square root of the perfect square.

Solution: Let us try to find $\sqrt{2050}$ by long division method.

	45
4	$\overline{20}\ \overline{50}$
	−16
85	450
	−425
	25

Here the remainder is 25. So, if we subtract 25 from 2050, we will get zero remainder.

It means if we subtract the remainder 25 from 2050 we will get a perfect square.

$\therefore$ The required perfect square is 2050 – 25 = 2025 and $\sqrt{2025} = 45$.

Example 3: What is the smallest number that should be added to 8860 to get a perfect square?

Solution: First let us try to find the square root of 8860 by long division method.

	94
9	$\overline{88}\ \overline{60}$
	−81
184	760
	−736
	24

The quotient is 94.

$\therefore\ 94^2 < 8860 < 95^2$

As we have to add a number to 8860 in order to get a perfect square, we should add the difference of $95^2 = 9025$ and 8860.

9025 – 8860 = 165

Thus, if 165 is added to 8860, we get a perfect square 9025, i.e. 95^2.

Example 4: Find the greatest five-digit number which is a perfect square.

Solution: The greatest five-digit natural number is 99999. We try to find its square root by long division method.

	316
3	$\overline{9}\ \overline{99}\ \overline{99}$
	−9
61	0 99
	−61
626	3899
	−3756
	143

Here the remainder is 143.

$\therefore$ The greatest five-digit number which is a perfect square is 99999 – 143 = 99856.

Example 5: Find the smallest six-digit number which is a perfect square.

Solution: The smallest six-digit number is 100000. We try to find its square root by long division method.

	316
3	$\overline{10}\ \overline{00}\ \overline{00}$
	−9
61	100
	−61
626	3900
	−3756
	144

Here, the quotient is 316. Square of 316 is a five-digit number.

$316^2 < 100000 < 317^2$

Where, $317^2 = 100489$

$\therefore$ 100489 is the smallest six-digit number which is a perfect square.

Example 6: In a charity organisation, a fund raising event was organised. For this, chairs had to be laid out in an equal number of rows and columns. If there are 2091 chairs in the stadium, find how many more chairs need to be brought in to complete the sitting arrangement?

Solution: As chairs are to be laid out in an equal number of rows and columns, number of chairs should be a perfect square.

That means we need to find the smallest number that should be added to 2091 to get a perfect square.

We shall find the square root of 2091.

	45
4	$\overline{20}\,\overline{91}$
	– 16
8$\underline{5}$	491
	– 425
	66

We get 66 as the remainder and 45 as the quotient which suggests $45^2 < 2091 < 46^2$.

$\therefore$ Number of more chairs needed = $46^2 - 2091 = 2116 - 2091 = 25$.

Hence, 25 chairs need to be brought in.

ENRICHMENT

For a perfect square having n digits the corresponding square root will have $\frac{n}{2}$ digits, if n is even and $\frac{n+1}{2}$ digits, if n is odd.

SQUARE ROOT OF DECIMALS

We are familiar with decimal numbers and studied them in detail in earlier classes.

RECALL

A decimal number has two parts – the number before decimal is called whole part and the number after decimal is called fractional part.

To find the square root of a decimal number, we shall follow the steps below:

Step 1: Start pairing the whole number part from right to left (as done for finding the square roots of numbers in the earlier section) and the fractional part from left to right. For example, to find square root of 148.84 we group the digits as follows:

$$\overline{1}\;\overline{48}.\overline{84}$$

If the decimal part contains odd number of digits, then suffix one zero at the end to make even number of digits. For example, for 148.84, we group the digits as $\overline{1}\;\overline{48}.\overline{84}\;\overline{00}$.

	12.2
1	$\overline{1}\;\overline{48}.\overline{84}$
	– 1
2$\underline{2}$	048
	– 44
24$\underline{2}$	484
	– 484
	0

Step 2: Perform long division method as usual to find the square root of the number given by placing the decimal in the quotient just before taking up the pair after decimal.

$\therefore \sqrt{148.84} = 12.2$

SOME EXAMPLES

Example 1: Find the square root of each of the following numbers by long division method:

(i) 24.01 (ii) 0.205209 (iii) 0.000529

Solution:

(i)

	4.9
4	$\overline{24}.\overline{01}$
	– 16
8<u>9</u>	801
	– 801
	0

(ii)

	0.453
4	$0.\overline{20}\,\overline{52}\,\overline{09}$
	– 16
8<u>5</u>	452
	–425
90<u>3</u>	2709
	– 2709
	0

(iii)

	0.023
0	$0.\overline{00}\,\overline{05}\,\overline{29}$
	– 00
02	005
	– 004
043	129
	– 129
	0

$\therefore \quad \sqrt{24.01} = 4.9$ $\qquad \therefore \quad \sqrt{0.205209} = 0.453$ $\qquad \therefore \quad \sqrt{0.000529} = 0.023$

Example 2: The area of a square plot is $101\frac{1}{400}$ m². Find the length of one side of the plot.

Solution: Area of the square plot = $101\frac{1}{400}$ m² $= \frac{40401}{400} = 101.0025$ m²

Therefore, the side of the square = $\sqrt{101.0025}$ m

	10.05
1	$\overline{1}\,\overline{01}.\overline{00}\,\overline{25}$
	– 1
2<u>0</u>	001
	– 0
20<u>0</u>	100
	– 0
200<u>5</u>	10025
	– 10025
	0

Hence, the side of the square plot = 10.05 m

ESTIMATING THE SQUARE ROOT

There are some practical situations, where we need only the estimated value of square roots of a number and not the exact value. In such cases, we estimate the square root of a number to the nearest whole number.

For example to estimate the square root of 450, we observe $20^2 = 400$ and $25^2 = 625$ as

$$400 < 450 < 625 \Rightarrow 20 < \sqrt{450} < 25.$$

To get a closer number, we observe here that

$$21^2 = 441 \text{ and } 22^2 = 484,$$

where 441 < 450 < 484.

$\therefore \quad 21 < \sqrt{450} < 22$

As 450 is much closer to 441 as compared to 484, therefore we can take estimated value of $\sqrt{450}$ as 21.

SOME EXAMPLES

Example 1: Estimate the value of $\sqrt{500}$ to the nearest whole number.

Solution: We know $20^2 = 400$ and $(25)^2 = 625$

$$20 < \sqrt{500} < 25$$

To get close estimate, we again observe

$22^2 = 484$ and $23^2 = 529$

$22 < \sqrt{500} < 23$

As 500 is closer to 484 as compared to 529, therefore we can take the estimated value of $\sqrt{500}$ as 22.

PRACTICE EXERCISE 3.4

(1) Find the square root of the following numbers by long division method:

(i) 119025 (ii) 9604 (iii) 11236 (iv) 61009 (v) 54756
(vi) 10816 (vii) 390625 (viii) 56169 (ix) 193600 (x) 90000

(2) Find the square root of the following decimal numbers by long division method:

(i) 39.69 (ii) 23.04 (iii) 12.25 (iv) 6.0516
(v) 66.4225 (vi) 17.64 (vii) 6.4009 (viii) 412.4961

(3) What least number should be subtracted from 1000 so that the resulting number becomes a perfect square?

(4) Find the least number which should be subtracted from 2361 to make it a perfect square.

(5) Find the least number which should be added to 256000 to make it a perfect square. Also find the square root of that perfect square.

(6) Find the greatest number of six digits which is a perfect square.

(7) Find the least number of seven digits which is a perfect square.

(8) Find the length of the side of a square whose area is 14.8225 m^2.

(9) Find the greatest member of the Pythagorean triplet whose two members are 36 and 77.

(10) In a right triangle ABC, right angled at B, AC = 29 cm and BC = 20 cm. Find the length of AB.

(11) Find the perimeter of a square whose area is twice the area of a field with area 269.12 m^2.

(12) For the Independence Day drill, 1450 students had to stand in such a manner that number of rows should be equal to number of columns. How many children were left out in this arrangement? How many rows were made?

(13) Find the number of digits in the square root of (i) 253009 and (ii) 1089.

(14) The area of a square plot is 40401 m^2. Find the length of side of the plot.

HOTS

1. Evaluate: $\sqrt{0.0196} + \sqrt{37.0881}$.
2. Find the number whose square is 0.256036.

PROJECT WORK

Take a S5, S10, S20, S50, S100, S500 notes each from your parents. Note down the serial numbers written on the note for each of the denominations (ignoring the letters). You will have a four-or five-or six-digit number for each of the notes. Check whether these numbers are perfect squares root or not. If yes, find their square roots. If not, estimate the square of these numbers.

MULTIPLE CHOICE QUESTIONS

(1) A perfect square cannot be written in the form:

(a) $4m + 3$. (b) $4m + 1$. (c) $3m$ (d) $3m + 1$.

(2) The value of $\sqrt{81}$ is:

(a) only 9. (b) only –9. (c) 9 or –9. (d) none of these.

(3) Square of which of the following numbers is odd?

(a) 284. (b) 300. (c) 495. (d) 172.

(4) The squares of which of the following number will not end with digit 9?

(a) 3373. (b) 4832. (c) 2867. (d) 1974.

(5) Square of 3257931 will end with digit:

(a) 1. (b) 2. (c) 3. (d) 4.

COMPREHENSIVE EXERCISE

(1) What is the sum of 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19?

(2) Using repeated subtraction, find whether 59 is a perfect square or not.

(3) Find the square root of the following using prime factorisation method:

(i) 7744 (ii) 42,025

(4) Find the least number which should be added to 18,220 to make a perfect square. Also find the square root of that perfect square.*

* For more practice questions refer to practice book.

CHAPTER CHECK-UP

- The square of a number is that number raised to the power of 2.
- Natural numbers which are the squares of some natural numbers are called perfect squares.
- A number ending with 2, 3, 7 or 8 is never a perfect square.
- Squares of even numbers are always even, and squares of odd numbers are always odd.
- If a number has 1 or 9 at units place, then its square has 1 in units place.
- If a number has 4 or 6 at units place, then its square has 6 in units place.
- Square of a number, ending with 5 also ends with 5.
- A square number can have only even number of zeros at the end.
- Perfect squares are always positive.
- Sum of first n odd natural numbers is n^2.
- Square of odd natural numbers can always be written as the sum of two consecutive positive numbers.
- Sum of any two consecutive triangular numbers is a square number.
- Between n^2 and $(n + 1)^2$ there are $2n$ non-square numbers.
- A number with n digits has either $(2n – 1)$ or $2n$ digits in its square.
- Every perfect square is of the form $3m$ or $3m + 1$.
- Every perfect square is of the form $4m$ or $4m + 1$.
- The difference of squares of two consecutive natural numbers is equal to the sum of those natural numbers.
- Square of a number can be found by column method, diagonal method or using expansion.
- Pythagorean triplets: If m is any natural number > 1, then $2m$, $m^2 – 1$ and $m^2 + 1$ form a Pythagorean triplet.
- Square root of a number n, denoted by $\sqrt{n}$, is the number which when multiplied by itself gives n as the product.

WEBLINKS:

www.flashcardexchange.com/tag/squares