PruQuest

MATHEMATICS

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PREFACE

Mathematics is widely used in our daily lives in various ways, which makes its study essential. The study of mathematics has to be focused on understanding the concepts and practising their applications to help cultivate reasoning, logic and thinking skills.

The PruQuest Mathematics series aims at the following:

- **Exploration** to emphasize on the discovery approach, that is, building knowledge by doing.
- **Understanding** to help learners actively build new knowledge from experience and prior knowledge, facilitating them to think, reason, analyse and articulate logically.
- **Application** to enable learners to structure the reality around them and prepare them better to encounter a wide variety of problems in life.

The unique approach of the series addresses all the demands of the National Curriculum Framework 2005 (NCF 2005). It also caters to the needs of different types of learners (Visual, Auditory and Kinaesthetic; VAK).

Key Features

- *Recall* to recapitulate the concepts which help build a chapter
- *Some Examples* to provide students with problem-solving strategies which would further enable them to tackle problems independently
- *Practice Exercises and Comprehensive Exercise* to apply the concepts learnt to problems and real-life situations
- *Enrichment* to enhance the conceptual knowledge
- *HOTS* to enable a learner think deeply about various mathematical ideas and develop mathematical skills such as reasoning and analytical thinking
- *Projects* to helps relate mathematics to everyday life and develop life skills
- *MCQs* to provide extra practice in the form of multiple-choice questions and prepare for different examinations
- *Chapter Check-Up* to provide definitions, key points and formulae to remember
- *Periodic Assessments and Sample Tests* to help in continuous evaluation at regular intervals and assess child's overall understanding

The series offers simple, wide-ranging and interesting learning material and thus aids in overall and complete development of the learner.

UNIT 1 – NUMBER SYSTEM

Chapter 1: Integers

Addition and Subtraction of Integers, Properties of Addition and Subtraction of Integers, Multiplication of Integers, Multiplication of Three Negative Integers, Multiplication of Four Negative Integers, Multiplication Properties of Integers, Division of Integers, Properties of Division of Integers

Chapter 2: Rational Numbers

Rational Numbers, Equivalent Rational Numbers, Positive and Negative Rational Numbers, Standard Form of Rational Number, Representation of Rational Numbers on Number Line, Comparison of Rational Numbers, Rational Numbers between two Rational Numbers, Operations on Rational Numbers

Chapter 3: Fractions

Addition and Subtraction of Fractions, Multiplication of Fractions, Multiplication of a Whole Number and a Fraction by a Fraction, Fraction as an Operator 'Of', Value of Products: Product of Two Proper and Two Improper Fractions; Product of a Proper and an Improper Fraction, Division of Fractions; Division of a Whole Number and a Fraction by Another Fraction, Multiplicative Inverse (Reciprocal), Division of Fraction by a Whole Number

Chapter 4: Decimals

Decimals, Addition and Subtraction of Decimals, Multiplication of Decimal Numbers by a Decimal Number and a Whole Number, Multiplication of a Decimal Number by 10, 100, 1000, 10000, Division of Decimals, Division of Decimals by 10, 100, 1000 and 10000, Division of a Decimal Number by a Whole Number, Division of a Decimal number by another Decimal Number

Chapter 5: Exponents and Powers

Exponents, Laws of Exponents, Decimal Number System, Standard Form

UNIT 2 – ALGEBRA

Chapter 6: Algebraic Expressions

Generation of an Algebraic Expression, Terms of an Algebraic Expression: Factors of a term; Coefficients of a term; Constant term; Degree of a term, Types of Terms, Types of Algebraic Expressions, Degree of an Algebraic Expression, Addition and Subtraction of an Algebraic Expression, Numerical Value of an Expression, Use of Algebraic Expressions: Rules for Perimeters; Rules for Areas; Rules for Number Patterns; Rules in Patterns Formed by Repetition of Shapes; Rules in Geometry

Chapter 7: Simple Equations

Forming an Equation, Solving an Equation using Rules of Balancing and Method of Transposing, Making Equations from Solution, Applications of Simple Equations

UNIT 3 – COMMERCIAL MATH

Chapter 8: Comparing Quantities

Equivalent Ratios, Percentage - Another Way of Comparing Quantities, Conversion of Fractional Numbers to Percentage, Conversion of Decimals to Percentage, Conversion of Percentage to Decimals, Use of Percentage, Increase or Decrease as Per Cent, Profit and Loss, Simple Interest

UNIT 4 – GEOMETRY

Chapter 9: Lines and Angles

Related Angles: Complimentary Angles; Supplementary Angles; Adjacent Angles; Vertically Opposite Angles; Linear Pair, Pair of Lines, Transversal, Angles formed by a Transversal, Checking for Parallel Lines

Chapter 10: Triangles and its Properties

Median of a Triangle, Altitude of a Triangle, Exterior Angle of a Triangle and its Properties, Angle Sum Property of a Triangle, Two Special Triangles: Equilateral and Isosceles Triangle, Triangle Inequality Property, Right-Angled Triangles and Pythagoras Property

Chapter 11: Congruence of Triangles

Congruence, Checking Congruency of Two Figures, Congruence of Line Segments, Congruence of Angles, Congruence of Triangles, Congruence of Squares, Congruence of Rectangles, Congruence of Circles, Criteria for Congruence of Triangles: Side-Side-Side (SSS) Congruence Criterion; Side-Angle-Side (SAS) Congruence Criterion; Angle-Side-Angle (ASA) Congruence Criterion; Right angle-Hypotenuse-Side (RHS) Congruence Criterion

Chapter 12: Practical Geometry

Parallel lines, Construction of a Line Parallel to a Given Line Using Ruler and Set Squares, Construction of a Line Parallel to a Given Line Using Ruler and Compasses: Through a point Outside the Given Line; At a given distance from the given line, Construction of Triangles, Construction of Triangle when Length of Three Sides are given (SSS Criterion), Construction of Triangle when Lengths of Two Sides and Measure of the Angle Between them are given (SAS Criterion), Construction of Triangle when Measures of Two of its Angles and the Length of the Side Included Between them are Given (ASA Criterion), Construction of Right Angled Triangle When the Length of One Leg and its Hypotenuse are Given (RHS Criterion)

Chapter 13: Symmetry

Line Symmetry, Lines of Symmetry for Regular Polygons, Rotational Symmetry, Rotational Symmetry of Regular Polygons, Rotational Symmetry of a Circle, Line Symmetry and Rotational Symmetry

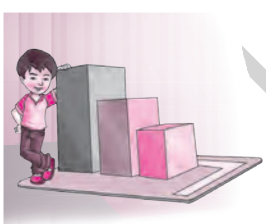
Chapter 14: Visualising Solid Shapes

Faces, Edges and Vertices, Nets of Solid, Drawing 3-D Shapes on Solids, Oblique Sketches, Isometric Drawing, Visualizing Solid Objects, Viewing Different Sections of a Solid, Shadow Play, View of Solids from Different Angle

UNIT 5 - MENSURATION

Chapter 15: Perimeter and area

Perimeter and Area of Square and Rectangle, Area of a Parallelogram, Area of a triangle, Circumference of a Circle, Area of a Circle, Conversion of Units, Area of a Path



UNIT 6 - STATISTICS

Chapter 16: Data Handling

Collecting and Organising Data, Measures of Central Tendency; Mean or Average, Range, Mode, Median, Bar Graphs, Double Bar Graphs, Construction and Interpretation of a Double Bar Graph, Chance and Probability

Problem Solving

Rule to Solve Problems

CONTENTS

Following are the symbols used in the book

Symbols used	∴	⇒	°	∠	Δ	≅
Meaning	Therefore	Implies	Degree	Angle	Triangle	Congruence

Chapter 1

Integers

INTRODUCTION

Whether we realise it or not, our life has been revolving around numbers since the day we were born. When a child is born, the parents become inquisitive to know how much their child weighs. When a child reaches at the age of four, he or she super excitedly waits to turn five. Throughout our school life, we go crazy over the percentage of marks we get in our examinations. No matter in what age we are, we are always eager to know how many gifts we received on our birthdays.

Numbers '0 to 9' play a very imperative role in our daily life. We see numbers on clocks and use them to know the time. There are numbers on doors and buildings for identification. The numbers from the set {0, 1, 2, 3, 4, ...}, that is, whole numbers, are called positive numbers. Situations dealing with positive numbers have their opposites dealing with negative numbers. A few examples from our daily life dealing with positive and negative numbers have been described in the following table:

Integers in Real Life

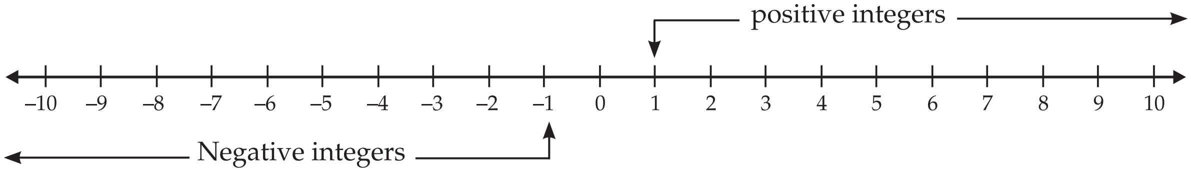
Positive	Negative
Temperature above zero (25°C)	Temperature below zero (–25°C)
Earning money (₹2000)	Having debt (–₹2,000)
Floors above the ground level (2nd floor, i.e. 2)	Basement car park levels (basement 2, i.e. –2)
Profit (₹250)	Loss (–₹250)
Above the speed limit (15 km/hr)	Below the speed limit (–15 km/hr)
Overweight (12 kg)	Underweight (–12 kg)

RECALL

(1) When collection of whole numbers is combined with the collection of negative numbers, the collection of **integers** is formed.

(2) The numbers 1, 2, 3, 4, 5, … are called positive integers, and the numbers –1, –2, –3, –4, –5,... are called **negative integers**. Zero is neither a positive nor a negative integer.

(3) A line on which numbers are marked at equal distances is called a **number line**. Below is a number line with a few integers marked on it. The integers go on endlessly in both the directions as shown in the following figure:



(4) Positive integers are greater than zero, while the negative integers are smaller than zero. Thus, every positive integer is greater than every negative integer.

Example: 5 > –2, 3 > –5 and so on.

(5) For any two integers on the number line, any number to the right is greater than the number to its left. Example: 7 > 3, 2 > –5, –4 > –9 and so on.

(6) There is no greatest integer and no smallest integer. However, the smallest positive integer is 1 and the greatest negative integer is –1.

(7) On the number line, when we move from left to right, the numbers are in ascending order and when we move from right to left, the numbers are in descending order.

(8) Absolute value of an integer is the numerical value of that integer, that is, value without the sign. The symbol for absolute value is | |.

Example: |–4| means absolute value or numerical value of –4, that is, 4.

ADDITION AND SUBTRACTION OF INTEGERS

A number line is a great tool for addition and subtraction of numbers. Thus, integers can be added or subtracted with the help of a number line.

In grade 6, we learnt a few rules that help us to add or subtract integers on a number line.

Rule 1: For adding a positive integer, we move to the right.

Rule 2: For adding a negative integer, we move to the left.

Rule 3: For subtracting a positive integer, we move to the left.

Rule 4: For subtracting a negative integer, we move to the right.

The following table describes the rules for addition of integers without using a number line:

Integers having the same sign	Integers having different signs
Step 1: Add the absolute values.	Step 1: Take absolute value of the integers and subtract the smaller absolute value from the larger absolute value. Alternatively, subtract the smaller numerical value from the larger.
Step 2: Keep the common sign.	Step 2: Keep the sign of the integer with larger numerical value.

We also learnt the concept of **additive inverse**. The additive inverse of a number is the number that when added to the given number gives zero. For example, additive inverse of 4 is –4 and additive inverse of –9 is 9.

Thus, to find the additive inverse of a number we keep the absolute value or the numerical value of the number same and reverse the sign i.e., positive to negative and negative to positive.

Note

(1) Additive inverse of a positive integer is a negative integer and that of a negative integer is a positive integer.

(2) Additive inverse of 0 is 0.

For subtraction of integers, we add the minuend and the additive inverse of the subtrahend.

Thus, for any two integers a and b,

$a - b = a$ + Additive inverse of b

$= a + (-b)$, and

$a - (-b) = a$ + Additive inverse of $-b$

$= a + b$

The number from which we subtract is called *minuend*.

The number which is subtracted is called *subtrahend*.

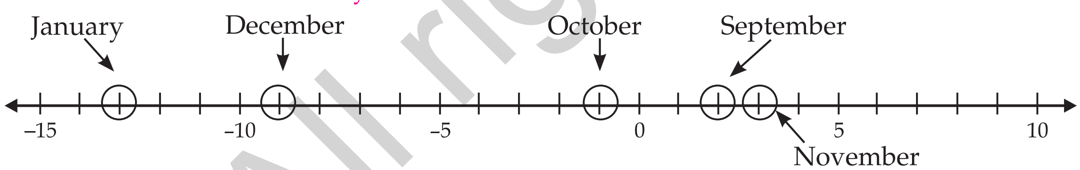
Note

We know that addition and subtraction are opposites of each other. So, if we are given the sum of two numbers, then each addend is equal to the difference of the sum and the other addend.

Thus, corresponding to the addition statement, 2 + 3 = 5, we have two subtraction statements: 3 = 5 – 2 and 2 = 5 – 3.

SOME EXAMPLES

Example 1: The following number line shows the average temperature in degree Celsius (°C) of a city on the five different months of a year.



(i) Write the average temperatures marked on the number line for the five months.

(ii) In which month the average temperature is more: January or October?

(iii) Arrange the months in ascending order of average temperature.

(iv) Find the difference between the average temperature in the month of November and December.

(v) Can we say that the average temperature in September and October taken together is less than the temperature in September? Is it also less than the temperature in October?

Solution: (i)

Months	January	September	October	November	December
Average Temperature (in °C)	–13	2	–1	3	–9

(ii) Average temperature in January = –13°C

Average temperature in October = –1°C

On the number line, –1 is to the right of –13.

Therefore, –1 > –13

Thus, the average temperature in October is more than that in January.

(iii) The months in ascending order of average temperature can be arranged as follows:
January, December, October, September, November.

(iv) Average temperature in November = 3°C
Average temperature in December = –9°C
The difference between the temperatures in the two months
= 3 – (–9) = 3 + 9 = 12°C [Since, $a - (-b) = a + b$]

(v) Average temperature in September and October taken together = 2 + (–1)
Here,
|2| – |–1| = 2 – 1 = 1 [Subtract the absolute values]
∴ 2 + (–1) = 1 [Keep the sign of integer with larger numerical value]
Thus, the average temperature in September and October taken together = 1°C.
This temperature is less than the temperature in the month of September and more than the temperature in the month of October.

Example 2: Arrange the given list of integers in descending order: 7, –5, 11, –2, –13, 14, 9.

Solution: First, consider the negative integers,–5, –2, –13.
Here, 2 < 5 < 13, therefore –2 > –5 > –13.
Now, consider the positive integers, 7, 11, 14, 9.

For any two integers a and b, $a > b$ implies $-a < -b$. Also, $a < b$ implies $-a > -b$. Example: 7 > 3; –7 < –3 and –5 < 9; 5 > –9.

The above integers in descending order will be arranged as 14, 11, 9, 7.
We know that positive integers are always greater than the negative integers.
Therefore, the given integers can be arranged in descending order as follows:
14, 11, 9, 7, –2, –5, –13

Example 3: For each of the following, add the integers:
(i) –72 + (–38) (ii) –108 + 54

Solution: (i) –72 + (–38) [Both are negative integers]
Here, |–72| + |–38| = 72 + 38 = 110 [Add the absolute values]
∴ –72 + (–38) = –110 [Keep common sign]
(ii) –108 + 54 [Integers given have opposite signs]
Here, |–108| – |54| = 108 – 54 = 54 [Subtract the absolute values]
∴ –108 + 54 = –54 [Keep the sign of larger]

Example 4: For each of the following, subtract the integers:
(i) –11 from 6 (ii) –27 from –19

Solution: (i) 6 – (–11) = 6 + 11 = 17 [Add additive inverse of –11 to 6]
(ii) –19 – (–27) = –19 + 27 [Add additive inverse of–27 to –19]
The integers –19 and 27 have opposite signs, thus we add the two integers as follows:
|27| – |– 19 | = 27 – 19 = 8 [Subtract the absolute values]
∴ –19 + 27 = 8 [Keep the sign of larger]
Thus, –19 – (–27) = 8

Example 5: Find the next three numbers in the pattern 11, 8, 5, 2, ___, ___, ___.

Solution: We observe that the numbers in the above pattern are in decreasing order. Also, there is a constant difference (that is, 3) between any two consecutive terms. Therefore, a term can be obtained by subtracting 3 from the previous term. Thus, 5th term = 4th term – 3 = 2 – 3 = –1,

6th term = 5th term – 3 = –1 – 3 = –4 and

7th term = 6th term – 3 = –4 – 3 = –7.

Thus, the next three terms in the pattern are –1, –4 and –7, respectively.

Example 6: Evaluate the following:

(i) 17 + (–6) + 9 + (–11) (ii) –47 + (–3) + 63 + (–16)

Solution:

(i) 17 + (–6) + 9 + (–11)

= {17 + 9} + {(–6) + (–11)}

= 26 + (–17)

= 9

(ii) –47 + (–3) + 63 + (–16)

= {–47 + (–3) + (–16)} + 63

= –{47 + 3 + 16} + 63

= –66 + 63

= –3

(1) Combine together positive and negative integers.
(2) Add the numerical values of all positive integers and put plus sign with the answer.
(3) Add the numerical values of all negative integers and put minus sign with the answer.
(4) Add the answers of step 2 and 3.

Example 7: The sum of two integers is 63. If one of the integer is –37, find the other.

Solution: The other integer = Sum of two integers— One of the integer = 63 – (–37) = 63 + 37 = 100.

Example 8: A vegetable seller made a profit of ₹53 on Monday, a loss of ₹17 on Tuesday and a loss of ₹7 on Wednesday. Find his net profit or loss in the three days.

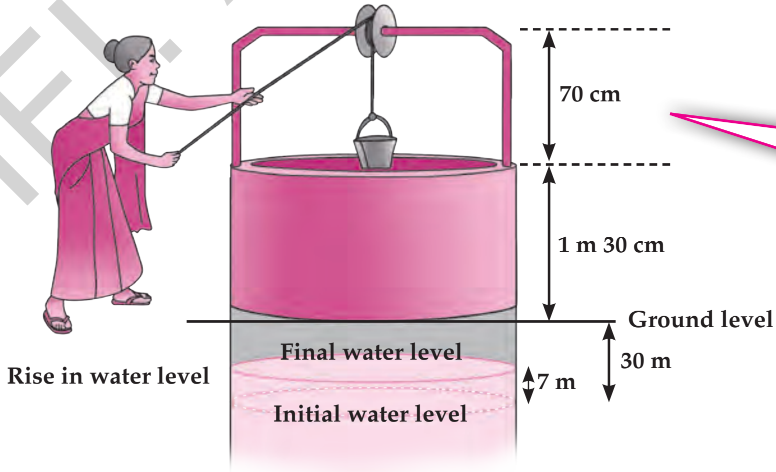
Solution: Profit means positive earning and loss means negative earning. Therefore, the seller's earning on Monday = ₹53, Tuesday = –₹17 and Wednesday = –₹7.

Thus, his net earnings in the three days = 53 + (–17) + (–7) = 53 + {(–17) + (–7)}

= 53 + {–(17 + 7)} = 53 + (–24) = 29

Hence, the vegetable seller made a profit of ₹29 in the three days.

Example 9: Water level in a well was 30 m below the ground level. During rainy season, rain water collected in different water tanks was drained into the well, and the water level rose 7 m above the previous level. The wall of the well is 1 m 30 cm high, and a pulley is fixed at a height of 70 cm. Ms Vitthal wants to draw water from the well. Find the minimum length of the rope that she should use.



It is always helpful to make a rough diagram for understanding the situation in the question.

Solution: The length of the rope to reach the underground water surface from the pulley

= Height of the pulley + Height of the wall + Water level below the ground level

Height of the pulley = 70 cm = 0.7 m

Height of the wall = 1 m 30 cm = 1.3 m

Final water level below the ground level = Initial water level below the ground level + Rise in water level

= (–30) m + 7 m = (–23) m

Thus, the length of the rope to reach the underground water surface from the pulley

= 0.7 m + 1.3 m – (–23) m = 2 m – (–23) m = 2 m + 23 m = 25 m

Thus, the minimum length of the rope thats he should use is 25 m.

HOTS

Fill each box with a correct integer:

(i) 7 – ☐ = 10 (ii) –23 + ☐ = 34 (iii) 8 – (–3) + ☐ = 13

(iv) (–5) – ☐ = 17

PRACTICE EXERCISE 1.1

(1) Arrange the following lists of integers in ascending order:

(i) 6, –6, –1, –4, 8, 5, 4 (ii) 11, –5, –4, 0, 3, 5 , 4, 1

(2) Arrange the following lists of integers in descending order:

(i) 8, –9, 4, 0, 3, 2, –2 (ii) –10, –7, 5, 4, 3, –3

(3) Find the sum of the following:

(i) 12 and –5 (ii) –23 and 19 (iii) –27 and – 44

(iv) –76 and 16 (v) 43 and –33

(4) Evaluate the following:

(i) –436 + 683 (ii) 1002 + (–14) (iii) –5463 + 234 (iv) (–6754) + (–786)

(5) Find the additive inverse of the following integers:

(i) –76 (ii) 786 (iii) –3452

(iv) 0 (v) 1

(6) Subtract the following:

(i) 44 from –56 (ii) –80 from 67 (iii) –55 from –78

(iv) 432 from 0 (v) –54 from 0

(7) Verify $a - (-b) = a + b$ for the following:

(i) $a = 15, b = 22$ (ii) $a = 124, b = 45$ (iii) $a = -56, b = 97$ (iv) $a = -23, b = 13$

(8) Evaluate the following:

(i) 23 + (–5) + 3 + (–8) (ii) (–90) + (–4) + 21 + (–9)

(iii) {78 + (–40)} + {(–67) + (–17)} (iv) {789 + (–1432)} – {(–1847) + 1760}

(9) Use >, < or = symbol as appropriate to make the following statements true:

(i) (–5) + (–7) _____ (–5) – (–7)
(ii) (–8) + 7 + (–9) _____ (–15) + 8 + (–6)
(iii) (–45) + 20 _____ 45 + (–20)
(iv) (–17) + 5 + (–34) _____ (–56) + 26
(v) 67 + 89 + (–532) _____ (–675) + 56 + 78

(10) Find the next number in the pattern: –62, –37, –12, _________.

(11) The sum of two integers is 45. If one of the integers is –23, determine the other.

(12) The difference of an integer from –56 is –5. Find the integer.

(13) The sum of two integers is –4. If one of the integers is –7, then find the additive inverse of the other integer.

(14) A monkey standing on the second stair goes up by seven more stairs. Which stair is he standing at now? Rahul placed a banana at the fourth stair. How many stairs is the monkey far from the stair having the banana?

(15) A bird is 40 m above the water level of a river. If the depth of the river is 55 m, then find the vertical distance between the bird and the bottom level of the river.

(16) A submarine was 2645 m below the sea level. If it descends 2400 m, what is its new position?

(17) A ship is at an elevation of 540 m above sea level when it starts. Later, elevation changes to 650 m, –467 m, 321 m, –132 m and 230 m. What is the elevation relative to the sea level in the end?

(18) There is a book balance race between Nida and Nidhi. The race was conducted in five parts. In the first part, Nida won by 15 seconds. In the second part, she lost by 2 minutes, won by 40 seconds in the third part, lost by 20 seconds in the fourth part and won by 12 seconds in the last part. Who won the race finally?

PROPERTIES OF ADDITION OF INTEGERS

CLOSURE PROPERTY

The sum of any two integers is always an integer, that is, integers are closed under addition.

If a and b are two integers, then $a + b$ is also an integer.

Example:

(1) The sum of integers 45 and –43 {45 + (–43)} is 2, which is an integer.

(2) The sum of integers (–12) and (–3) {–12 + (–3)} is –15, which is an integer.

COMMUTATIVE PROPERTY

Two integers can be added in any order, that is, addition is commutative for integers.

If a and b are two integers, then $a + b = b + a$.

Example: 7 + (–8) = –1 and (–8) + 7 = –1, therefore 7 + (–8) = (–8) + 7.

ASSOCIATIVE PROPERTY

When adding three or more integers, it does not matter which two are added first, that is, addition is associative for integers.

If a, b and c are three integers, then $(a + b) + c = a + (b + c)$.

Example: $\{52 + (-24)\} + 38 = 28 + 38 = 66$ and $52 + \{(-24) + 38\} = 52 + 14 = 66$

Thus, $\{52 + (-24)\} + 38 = 52 + \{(-24) + 38\}$

ADDITIVE IDENTITY

If we add zero to any integer or any integer to zero, the result is integer itself. So, we say 0 is the identity for integers under addition or 0 is the additive identity for integers.

If a is any integer, then $a + 0 = 0 + a = a$.

Example:

(1) $7 + 0 = 7 = 0 + 7$

(2) $0 + (-9) = (-9) = (-9) + 0$

ADDITIVE INVERSE

For any integer a, there exists its opposite $-a$ such that the sum of the two integers is zero. Integer a and $-a$ are called *opposites* or *negatives* or *additive inverse of each other*.

If a is any integer, then $a + (-a) = 0 = (-a) + a$.

Example: $3 + (-3) = 0 = (-3) + 3$

Thus, 3 and –3 are opposites or negatives or additive inverse of each other.

PROPERTIES OF SUBTRACTION OF INTEGERS

CLOSURE PROPERTY

The difference of any two integers is always an integer, that is, integers are closed under subtraction.

If a and b are two integers, then $a - b$ is also an integer.

Example:

(1) When we subtract (–38) from 67, we get 105; which is an integer.

(2) When we subtract (–25) from –13, we get 12; which is an integer.

COMMUTATIVE PROPERTY

The difference of any two integers depends on the order in which they are subtracted, that is, subtraction is not commutative for integers.

If a and b are two integers, then $a - b \neq b - a$.

Example:

(1) $23 - 17 = 6$ and $17 - 23 = -6$, therefore $23 - 17 \neq 17 - 23$

(2) $57 - (-38) = 95$ and $(-38) - 57 = -95$, therefore $57 - (-38) \neq (-38) - 57$

ASSOCIATIVE PROPERTY

When subtracting three or more integers, the difference of the integers depends on which of the two integers are subtracted first, that is, subtraction is not associative for integers.

> If a, b and c are three integers, then $(a - b) - c \neq a - (b - c)$.

Example: $\{1-(-9)\} - 5 = 10 - 5 = 5$, $1 - (-9 - 5) = 1 - (-14) = 15$

Thus, $\{1-(-9)\} - 5 \neq 1 - (-9 - 5)$

IDENTITY UNDER SUBTRACTION

Let a be any integer. There does not exist any integer x for which $a - x = x - a = a$, that is, there is no integer which can display the characteristic of an identity element for integers under subtraction.

INVERSE UNDER SUBTRACTION

Since, the identity for subtraction doesn't exist, the question of inverse under subtraction does not arise.

PROJECT WORK

Managing Your Pocket Money!

Below is a sample table that will help you prepare a record for managing your pocket money well.

Month: January   Pocket Money: ₹500

Description (money spent/ money received)	Amount	Balance
Purchased stationary worth ₹56	–₹56	₹444 (500 –56)
Gift money of ₹100 received from grandmother	+₹100	₹544 (444 + 100)
...	...	...
Total		₹ ______

Fill in the details of the money you spent (e.g., you purchased stationary worth ₹56), that is, the amount is –₹56 and the money you received (e.g., gift money of ₹100 received from grandmother on a festival or birthday), that is, amount is +₹100.

Calculate the balance (i.e., savings at the end of the month). The initial amount is the pocket money that you get. To know the balance we subtract the money spent and add the money received.

How important do you think is saving money? Do you think you could have saved more money by avoiding a few spends?

SOME EXAMPLES

Example 1: Write a pair of integers whose sum is

(i) a negative integer.

(ii) an integer smaller than only one of the integers.

(iii) zero.

Solution: (i) Sum of two negative integers is a negative integer.

One such pair of integers will be (–5, –3) as {–5 + (–3)} = –8, which is a negative integer.

(ii) Let the pair of integers be (–2, 6), then (–2 + 6) = 4, which is an integer.

Also, 4 > –2 and 4 < 6

Hence, the sum is an integer which is smaller than only one of the integers.

(iii) The sum of a number and its additive inverse is zero. One such pair of integers will be (1, –1) as 1 + (–1) = 0.

Example 2: Write a negative and a positive integer such that their difference is –4.

Solution: Let the negative integer and the positive integer be –2 and 2 respectively.

When we subtract 2 from (–2), we get {(–2) – 2}, that is, –4.

Hence, the required pair of integers is (–2, 2).

Example 3: Fill in the blanks to make the statements true. Also, name the property used.

(i) 3 + (–2) = ______ + 3

(ii) –27 + ______ = –27

(iii) 51 + (–51) = ______

Solution:

(i) 3 + (–2) = __–2__ + 3, by using commutative property of addition for integers.

(ii) –27 + __0__ = –27, by using the property of additive identity of integers.

(iii) 51 + (–51) = __0__, by using the property of additive inverse of integers.

PRACTICE EXERCISE 1.2

(1) Write down a pair of integers whose sum is

(i) –3.

(ii) zero.

(iii) an integer smaller than both the integers.

(iv) an integer greater than both the integers.

(v) a positive integer.

(2) Write a pair of integers such that when second integer is subtracted from the first integer, the result is

(i) an integer smaller than both the integers.

(ii) an integer greater than only one of the integers.

(iii) –12.

(iv) 19.

(v) zero.

(vi) a positive integer.

(vii) a negative integer.

(viii) smaller than both the integers.

(3) Fill in the blanks to make the statements true. Also, name the property used.

(i) (–7) + (–5) = _____ + (–7)

(ii) (–67) + _____ = –67

(iii) 14 + (–14) = _____

(iv) 19 + _____ = 19

(v) 5 + {(–9) + (12)} = (5 + 12) + _____

(vi) 8 + ________ = 0

(4) With the help of a suitable example, show that the subtraction is non-commutative and non-associative for integers.

(5) Verify that $(a + b) + c = a + (b + c)$ for the following values of a, b and c:

(i) $a = 23$, $b = 55$ and $c = –87$ (ii) $a = 45$, $b = –12$ and $c = –52$ (iii) $a = –18$, $b = 0$ and $c = 17$

MULTIPLICATION OF INTEGERS

In earlier grades, we learnt multiplication of whole numbers, that is, multiplication of positive integers. We now extend our learning to the multiplication of negative integers and multiplication of a positive integer (a whole number) with a negative integer.

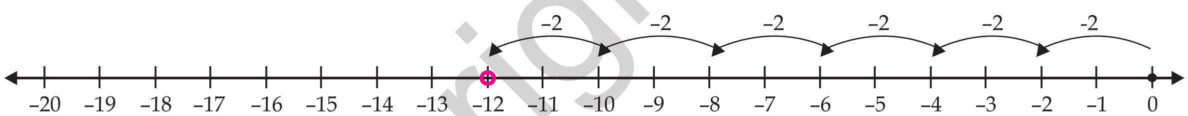
MULTIPLICATION OF INTEGERS WITH UNLIKE SIGNS

Let us evaluate 6 × (–2). We know that multiplication is the repeated addition of the number to be multiplied (multiplicand).

Thus, 6 × (–2) can be written as (–2) + (–2) + (–2) + (–2) + (–2) + (–2).

So, we have expressed the product of a positive integer and a negative integer as the sum of negative integers. Let us find this sum by using the number line. Start from 0 and move two steps to the left.

Repeating the above process six times, we reach –12, which is the required sum.



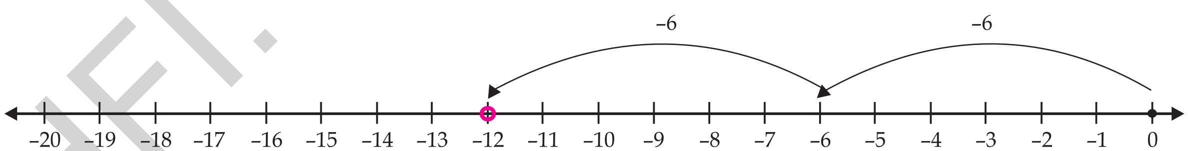
From the above number line, we can see (–2) + (–2) + (–2) + (–2) + (–2) + (–2) = 6 × (–2) = –12.

Now, –12 can also be written as –(6 × 2) which is the same as –(|6| × |–2|).

Thus, 6 × (–2) = –(|6| × |–2|) = –(6 × 2) = –12

Let us now evaluate 2 × (–6).

Again, we write 2 × (–6) as (–6) + (–6).



To do the addition on the number line, we start from 0 and move six steps to the left two times. This brings us to –12, which is the required sum.

Thus, we see (–6) + (–6) = 2 × (–6) = –12.

Observe that –12 can also be written as –(2 × 6) which is the same as –(|2| × |–6|).

Thus, 2 × (–6) = –(|2| × |–6|) = –(2 × 6) = –12

Let us now find the product, (negative integer) × (positive integer).

Evaluate –6 × 2.

To do so, observe the pattern as shown below:

6	×	2	=	12	
5	×	2	=	10	= 12 – 2
4	×	2	=	8	= 10 – 2
3	×	2	=	6	= 8 – 2
2	×	2	=	4	= 6 – 2
1	×	2	=	2	= 4 – 2
0	×	2	=	0	= 2 – 2
–1	×	2	=	–2	= 0 – 2
–2	×	2	=	–4	= –2 – 2
–3	×	2	=	–6	= –4 – 2
–4	×	2	=	–8	= –6 – 2
–5	×	2	=	–10	= –8 – 2
–6	×	2	=	–12	= –10 – 2
Negative integer	×	Positive integer		Negative integer	

Each number is obtained by subtracting 2 from the previous number.

Therefore, –6 × 2 = –12.

From the above examples, we observe that 6 × (–2) = –12 = (–6) × 2.

Likewise, (–7) × 3 = –21 = 7 × (–3) and (–5) × 8 = –40 = 5 × (–8).

In general, we can say that:

> To find the product of two integers with unlike signs, find the product of their numerical values (absolute values) and assign a minus sign to the product.

Thus, for any two positive integers *a* and *b*, $a \times (-b) = (-a) \times b = -(a \times b)$

MULTIPLICATION OF INTEGERS WITH LIKE SIGNS

In this section, we will learn how to find the product of two negative integers.

Let us evaluate (–4) × (–3). To do so, observe the following pattern:

4	×	(–3)	=	–12	
3	×	(–3)	=	–9	= –12 + 3
2	×	(–3)	=	–6	= –9 + 3
1	×	(–3)	=	–3	= –6 + 3
0	×	(–3)	=	0	= –3 + 3
–1	×	(–3)	=	3	= 0 + 3
–2	×	(–3)	=	6	= 3 + 3
–3	×	(–3)	=	9	= 6 + 3
–4	×	(–3)	=	12	= 9 + 3
Negative integer	×	Negative integer		Positive integer	

Each number is obtained by adding 3 to the previous number.

Therefore, (–4) × (–3) = 12.

Observe that, 12 = 4 × 3 = |–4| × |–3|.

From the above expression, we observe that when both the multiplier and the multiplicand are negative integers, then their product is positive. Therefore, the product of two negative integers is a positive integer.

In general, we can say that:

> To find the product of two negative integers, find the product of their numerical values (absolute values) and assign a plus sign to the product.

Thus, for any two positive integers a and b, $(-a) \times (-b) = a \times b$

Thus, we have (–12) × (–13) = 12 × 13 = 156.

Similarly, (–15) × (–7) = 105.

The following table shows the rules for multiplication of integers:

Integers with unlike signs	Integers with like signs
Positive × Negative gives Negative	Positive × Positive gives Positive
Negative × Positive gives Negative $a \times (-b) = (-a) \times b = -(a \times b)$	Negative × Negative gives Positive $(-a) \times (-b) = a \times b$

Note

For any integer a,

$(-1) \times a = -(1 \times a) = -a$

$(-1) \times (-a) = 1 \times a = a$

No matter how many positive integers we multiply, the product will always be positive. Let us see what happens to the sign of the product if we multiply more than two negative integers.

MULTIPLICATION OF THREE NEGATIVE INTEGERS

From the previous section, we know that,

Negative integer × Negative integer = Positive integer

∴ (Negative integer × Negative integer) × Negative integer

= Positive integer × Negative integer

Now, a positive integer (the product of the first two negative integers) is to be multiplied with a negative integer, which gives a negative integer.

∴ (Negative integer × Negative integer) × Negative integer = Negative integer

Example: {(–3) × (–5)} × (–7) = 15 × (–7) [Negative × Negative = positive]

= – (15 × 7) = –105 [Positive × Negative = Negative]

MULTIPLICATION OF FOUR NEGATIVE INTEGERS

We have just seen that product of three negative integers is negative. Now, if this product is multiplied with a fourth negative integer, the result is a positive integer because product of two negative integers is a positive integer.

Example: {(–2) × (–4)} × (–7) × (–5) = {8 × (–7)} × (–5) = (–56) × (–5) = 280

Similarly, by multiplying five negative integers we will get a negative integer.

So, in general, if the number of negative integers in a product is *even*, then the product is a *positive integer*, and if the number of negative integers in a product is *odd*, then the product is a *negative integer*.

Note

(–1) × (–1) = +1

(–1) × (–1) × (–1) = –1

(–1) × (–1) × (–1) × (–1) = +1

(–1) × (–1) × (–1) × (–1) × (–1) = –1

Observe that, if the integer (–1) is multiplied even number of times, the product is +1, and if the integer (–1) is multiplied odd number of times, the product is –1.

SOME EXAMPLES

Example 1: Find: 13 × (–7)

Solution: 13 × (–7) = – (13 × 7) [Positive × Negative, gives Negative]

= –91

Example 2: Evaluate: (–15) × (–11)

Solution: (–15) × (–11) = (15 × 11) [Negative × Negative, gives Positive]

= 165

Example 3 Find: (–9) × (–5) × (–6) × (–3)

Solution: (–9) × (–5) × (–6) × (–3) = [{(–9) × (–5)} × –6] × (–3)

= [45 × –6] × (–3)

= –270 × (–3) = 810

Example 4: What will be the sign of the product if we multiply together

(i) 6 negative integers and 13 positive integers?

(ii) (–1), 2*n* times, where *n* is a natural number?

Solution: (i) The number of negative integers in the product is even, therefore the product of these negative integers is a positive integer.

The product of positive integers is always positive.

Thus, the product of 6 negative integers and 13 positive integers is positive.

(ii) For any natural number *n*, 2*n* being multiple of 2 is an even number. Since, (–1) is multiplied even number of times, the product is +1, which is positive.

Example 5: Determine the integer whose product with –1 is 54.

Solution: The product is positive and one of the factors is negative, therefore the other factor has to be negative as negative × negative = positive. Also, the numerical value of one of the factors is 1; therefore, to get the product as 54, the numerical value of the other factor must be 54.

Thus, the required integer is –54.

MULTIPLICATION PROPERTIES OF INTEGERS

CLOSURE PROPERTY

The product of any two integers is always an integer, that is, integers are closed under multiplication.

> If a and b are any two integers, then $a \times b$ is also an integer.

Example:

(1) $(-9) \times 3 = -27$, which is an integer. (2) $(-12) \times (-11) = 132$, which is an integer.

COMMUTATIVE PROPERTY

Two integers can be multiplied in any order, that is, multiplication is commutative for integers.

> If a and b are two integers, then $a \times b = b \times a$.

Example:

(1) $12 \times (-8) = -96$ and $(-8) \times 12 = -96$, therefore $12 \times (-8) = (-8) \times 12$

(2) $(-7) \times (-6) = 42$ and $(-6) \times (-7) = 42$, therefore $(-7) \times (-6) = (-6) \times (-7)$

ASSOCIATIVE PROPERTY

When multiplying three or more integers, it does not matter which two are multiplied first, that is, multiplication is associative for integers.

> If a, b and c are three integers, then $a \times (b \times c) = (a \times b) \times c$.

Example: $3 \times \{(-5) \times 2\} = 3 \times (-10) = -30$ and $\{3 \times (-5)\} \times 2 = (-15) \times 2 = -30$,
Therefore, $3 \times \{(-5) \times 2\} = \{3 \times (-5)\} \times 2$

MULTIPLICATIVE IDENTITY

If we multiply any integer by 1, the result is the integer itself. So, we say 1 is the identity for integers under multiplication or 1 is the multiplicative identity of integers.

> If a is any integer, then $a \times 1 = 1 \times a = a$.

Example:

(1) $7 \times 1 = 1 \times 7 = 7$ (2) $(-2) \times 1 = 1 \times (-2) = -2$

MULTIPLICATIVE INVERSE

Two integers are called multiplicative inverse of each other, if their product gives the multiplicative identity, that is, 1. In multiplication, this is not possible for any integer except 1.

For any integer a, its multiplicative inverse is $\frac{1}{a}$ as $a \times \frac{1}{a} = 1$. However, $\frac{1}{a}$ is not an integer except for 1.

Therefore, for all integers except 1, the multiplicative inverse does not exist.

Note

The only integer whose multiplicative inverse exists is 1 and the multiplicative inverse is 1 itself. This is because $1 \times 1 = 1$.

DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION

For any three integers, multiplication distributes over addition, that is, the product of the first integer and the sum of the other two is equal to the sum of the product of the first and the second integer and the product of the first and the third integer.

If a, b and c are three integers, then $a \times (b + c) = (a \times b) + (a \times c)$.

Example: 9 × {6 + (–2)} = 9 × 4 = 36 and 9 × 6 + {9 × (–2)} = 54 + (–18) = 54 – 18 = 36

∴ 9 × {6 + (–2)} = (9 × 6) + {9 × (–2)}

DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER SUBTRACTION

For any three integers, multiplication distributes over subtraction, that is, the product of the first integer with the difference of the other two is equal to the difference between the product of the first and the second integer and the product of the first and the third integer as mentioned here:

If a, b and c are any three integers, then $a \times (b - c) = (a \times b) - (a \times c)$.

Example: 11 × (8 – 5) = 11 × 3 = 33 and (11 × 8) – (11 × 5) = 88 – 55 = 33

Thus, 11 × (8 – 5) = (11 × 8) – (11 × 5)

MULTIPLICATION BY ZERO

Product of any integer with zero gives zero as the result.

If a is any integer, then $a \times 0 = 0 \times a = 0$.

Example:

(1) 9 × 0 = 0, 0 × 9 = 0

(2) (–11) × 0 = 0, 0 × (–11) = 0

SOME EXAMPLES

Example 1: Verify that 18 × {7 + (–3)} = (18 × 7) + {18 × (–3)}.

Solution: LHS = 18 × {7 + (–3)} = 18 × 4 = 72

RHS = (18 × 7) + {18 × (–3)} = 126 + (–54) = 126 – 54 = 72

∴ LHS = RHS

Hence, verified.

Example 2: Find the product of the following integers by using suitable properties.

(i) 15 × (–25) × (–4) × (–7) (ii) 8 × (–53) × (–125)

(iii) (–41) × 102 (iv) (–17) × 29

Solution: (i) 15 × (–25) × (–4) × (–7)

To make multiplication easier, we group the numbers by using the commutative and associative properties of multiplication of integers.

Thus, 15 × (–25) × (–4) × (–7) = [{15 × (–25)} × (–4)] × (–7)

= [15 × {(–25) × (–4)}] × (–7) [Using associative property]

= (15 × 100) × (–7) = 1500 × (–7) = –10500

(ii) 8 × (–53) × (–125)

= {8 × (–53)} × (–125)

= {(–53) × 8} × (–125) [Using commutative property]

= (–53) × {8 × (–125)} [Using associative property]

= (–53) × (–1000) = 53000

(iii) (–41) × 102

= (–41) × (100 + 2) [Writing 102 as 100 + 2]

= {(–41) × 100} + {(–41) × 2}

[Using distributive property of multiplication over addition]

= –4100 + (–82) = –4182

(iv) (–17) × 29

= (–17) × (30 – 1) [Writing 29 as 30 – 1]

= (–17 × 30) – (–17 × 1)

[Using distributive property of multiplication over subtraction]

= –510 – (–17) = –510 +17 = –493

Example 3: In a class test containing 10 questions, 5 marks are awarded for every correct answer, (–2) marks for every incorrect answer and 0 for questions not attempted. Find the scores of the students, if:

(i) Mrinal has given six correct and four incorrect answers.

(ii) Rupali has attempted all the questions, but only five of her answers are correct.

(iii) Harshit has given three correct and four incorrect answers out of seven questions he attempted.

Solution: (i) Marks given for one correct answer = 5.

So, marks given for six correct answers = 5 × 6 = 30.

Marks given for one incorrect answer = –2.

So, marks given for four incorrect answers = (–2) × 4 = –8.

Therefore, Mrinal's total score = 30 + (–8) = 22.

(ii) Marks given for one correct answer = 5.

So, marks given for five correct answers = 5 × 5 = 25.

Marks given for one incorrect answer = (–2).

So, marks given for five incorrect answers = (–2) × 5 = –10.

Therefore, Rupali's total score = 25 + (–10) = 15.

(iii) Marks given for one correct answer = 5.

So, marks given for three correct answers = 5 × 3 = 15.

Marks given for one incorrect answer = –2.

So, marks given for four incorrect answers = (–2) × 4 = –8.

Marks for questions not attempted = 0

So, marks given for 3 (= 10 – 7) questions not attempted = 0 × 3 = 0

Therefore, Harshit's total score = 15 + (–8) + 0 = 7

Example 4: A multistorey building has 25 floors above the ground level each of height 5 m. It also has three floors in the basement each of height 5 m. A lift in the building moves at a rate of 1 m/s. If a man starts from the 10th floor above the ground, how long does it take him to reach at the 2nd floor of the basement?

Solution: Height of each floor in the building = 5 m

Distance from the ground level to the 10th floor = 10 × 5 m = 50 m

When the lift goes down the ground level, the distance covered by it is represented by a negative integer.

Therefore, distance from the ground level to the 2nd floor of the basement

= 2 × (–5 m) = –10 m

∴ Distance between the 10th floor and the 2nd floor of the basement

= 50 m – (–10 m) = 50 m + 10 m = 60 m

It is given that the time taken by the lift to travel a distance of 1 m = 1 s.

So, the time taken by the lift to travel a distance of 60 m = 1 × 60 = 60 s = 1 min

Thus, it will take the lift 1 minute to reach at the 2nd floor of the basement from the 10th floor.

PRACTICE EXERCISE 1.3

(1) Find the product in each of the following:

(i) 23 × 7 (ii) (–9) × 1 (iii) (–87) × (–1)

(iv) (–67) × 0 (v) (–21) × (–26) (vi) (–120) × (35)

(vii) (–125) × 8 (viii) 4 × 6 × (–8) (ix) 9 × (–12) × (–14)

(x) (–5) × (–2) × (–7) (xi) (–7) × (–12) × 15 (xii) (–20) × (–24) × (–6)

(xiii) 4 × (–9) × 16 (xiv) (–1) × (–3) × (–7) × 13 (xv) 12 × (–5) × (–17) × (–1)

(2) Verify the following products:

(i) 6 × (–43) = (–43) × 6

(ii) 7 × {8 × (–4)} = (–4) × (7 × 8)

(iii) 31 × {11 + (–3)} = 31 × 11 + 31 × (–3)

(iv) (–52) × {(–4) + (–17)} = {(–52) × (–4)} + {(–52) × (–17)}

(3) What will be the sign of the product if we multiply the following?

(i) 12 negative integers and 4 positive integers

(ii) 18 positive integers and 13 negative integers

(iii) 17 negative integers and 21 negative integers

(iv) 23 positive integers and 22 positive integers

(4) Determine the integer whose product with (–1) is

(i) –98. (ii) 86. (iii) 0. (iv) –34.

(5) Fill in the blanks and name the property used in each of the following:

(i) (–8) × (–143) = ______ × (–8)

(ii) {(–15) × 21} × 29 = (21 × 29) × ______

(iii) (–37) × {61 + (–53)} = (–37) × 61 + ____ × (–53)

(iv) (–8765) × _____ = (–8765)

(v) (–4532) × 0 = _____

(6) Find the products by using suitable property.

(i) (–64) × (–43) + (–43) × (–32)

(ii) 20 × (–52) + 20 × (–52)

(iii) 49 × (–25) + (–25) × (–39)

(iv) 27 × (–16) + (–16) × 23

(v) 142 × (–25) + (–142) × 75

(vi) 45 + (–59) × (–45)

(vii) 5678 × (–6) + (–5678) × 94

(viii) 9 × 54 – 9 × 4

(ix) 23 × (–25) × (–4) × (–10)

(x) 765 × 1565 – 565 × 765

(7) Find that which one is greater?

(i) (4 + 9) × 10 or 4 + 9 × 10

(ii) (8 – 12) × 10 or 8 – 12 × 10

(iii) {(–6) – 8} × (–11) or (–6) – 8 × (–11)

(8) Fill in the blanks with an integer to make it a true statement.

(i) (–6) × ____ = –42

(ii) 8 × ____ = – 48

(iii) ____ × (–9) = 63

(iv) ____ × (–13) = –182

(9) A monkey can climb 2 m in 1 minute. After 15 minutes, how high can it reach?

(10) A stationary shopkeeper earns a profit of ₹2 by selling a pen and incurs a loss of ₹3 by selling a notebook.

(i) If he sold 25 pens and 15 notebooks on a day, what would be his net profit or loss on that day?

(ii) What is the number of pens he must sell on a day to have neither profit nor loss if the number of notebooks sold on that day is 50?

(11) In a class test containing 20 questions, 5 marks are awarded for every correct answer and (–3) marks for every incorrect answer and 0 for questions not attempted.

(i) Bhanu gets 12 correct and 6 incorrect answers, and she doesn't attempt 2 questions. What is her score?

(ii) Naresh gets 10 correct answers and 10 incorrect answers. What is his score?

(iii) Shalini gets 5 correct and 9 incorrect answers out of the 14 questions that she attempted. What is her score?

(12) Write a pair of integers whose product is (–14) and whose difference is 15 or –15.

(13) Write a pair of integers whose product is (–24) and there lies 9 integers between them (excluding the integers in the pair).

(14) Write two integers whose product is

(i) smaller than both the integers.

(ii) greater than both the integers.

(15) If Δ is an operation such that for integers a and b, we have $a \Delta b = a \times a + b \times b - a \times b$, then find (–6) Δ 5.

(16) If Δ is an operation such that for integers a and b, we have $a \Delta b = (a \times b) + (b \times b) - (3 \times a \times b) + \{b \times b \times (-a)\}$, then find

(i) 4 Δ (–6) and (–6) Δ 4.

(ii) (–6) Δ (–8) and (–8) Δ (–6).

DIVISION OF INTEGERS

We know that multiplication and division are opposite of each other. Consider an example of multiplication of 23 and 4. That is, 23 × 4 = 92.

This implies that, 92 ÷ 4 = 23 and 92 ÷ 23 = 4. So, for every multiplication sentence of whole numbers, there are two division sentences.

Following are a few examples of multiplication sentences and their corresponding division sentences.

Multiplication sentence	Corresponding division sentences	
5 × (–7) = (–35)	(–35) ÷ (–7) = 5,	(–35) ÷ 5 = (–7)
(–3) × 9 = (–27)	(–27) ÷ 9 = –3,	(–27) ÷ (–3) = 9
(–8) × (–9) = 72	72 ÷ (–9) = –8,	72 ÷ (–8) = –9
(–2) × (–6) = 12	12 ÷ (–2) = –6,	12 ÷ (–6) = –2
24 × 3 = 72	72 ÷ 24 = 3,	72 ÷ 3 = 24

From the above table, we observe the following:

(–35) ÷ 5 = (–7)
(–27) ÷ 9 = (–3)

Negative integer ÷ Positive integer = Negative integer

So, when we divide a negative integer by a positive integer, we divide them as whole numbers, that is, we divide their numerical values and then put a minus sign (–) before the quotient.

Thus, the quotient obtained on dividing a negative integer by a positive integer is a negative integer.

We also observe

72 ÷ (–8) = –9
72 ÷ (–9) = –8
12 ÷ (–6) = –2
12 ÷ (–2) = –6

Positive integer ÷ Negative integer = Negative integer

So, when we divide a positive integer by a negative integer, we first divide them as whole numbers, that is, we divide their numerical values and then put a minus sign (–) before the quotient.

Thus, the quotient obtained on dividing a positive integer by a negative integer is a negative integer.

In fact, the above two results can be generalized as follows:

> For any two integers a and b, $a \div (-b) = -(a \div b) = (-a) \div b$ where $b \neq 0$.

Observe the division statements as follows:

(–35) ÷ (–7) = 5
(–27) ÷ (–3) = 9

Negative integer ÷ Negative integer = Positive integer

So, we can say that when we divide a negative integer by a negative integer, we first divide them as whole numbers, that is, divide their numerical values and then put a plus sign (+) before the quotient.

Thus, the quotient obtained on dividing a negative integer by a negative integer is a positive integer.

In general,

For any two positive integers a and b, $(-a) \div (-b) = a \div b$, where $b \neq 0$.

Following are the general rules for division of integers:

Positive	÷	Positive	=	Positive
Positive	÷	Negative	=	Negative
Negative	÷	Positive	=	Negative
Negative	÷	Negative	=	Positive

Precisely, if dividend and divisor are of same sign, quotient is positive and if dividend and divisor are of opposite sign, quotient is negative.

DIVISIONS INVOLVING ZERO

DIVISION BY ZERO

Just like whole numbers, dividing any integer by zero is meaningless. For any integer a, $(a \div 0)$ or $\frac{a}{0}$ is not defined.

ZERO DIVIDED BY ANY INTEGER

Zero divided by any integer is always zero. For any integer a, $(0 \div a)$ or $\frac{0}{a}$ is 0.

PROPERTIES OF DIVISION OF INTEGERS

CLOSURE PROPERTY

The quotient of any two integers may not be an integer, that is, integers are not closed under division.

Example: On dividing integers –9 and 4, we get $(-9) \div 4 = -\frac{9}{4}$, which is not an integer.

COMMUTATIVE PROPERTY

Changing the order of dividend and divisor changes the answer of division, that is, division is not commutative for integers. If a and b are two integers, then $a \div b \neq b \div a$.

Example: $\frac{-5}{10} = -\frac{1}{2}$ and $\frac{10}{-5} = -2$, therefore, $-5 \div 10 \neq 10 \div (-5)$

ASSOCIATIVE PROPERTY

Division is not associative for integers. If a, b and c are three integers, then $(a \div b) \div c \neq a \div (b \div c)$.

Example: $\{48 \div (-8)\} \div 2 = (-6) \div 2 = -3$ and $48 \div (-8 \div 2) = 48 \div (-4) = -12$, therefore, $\{48 \div (-8)\} \div 2 \neq 48 \div (-8 \div 2)$

IDENTITY UNDER DIVISION

Let a be any integer. There does not exist any integer x for which $a \div x = x \div a = a$, that is, there is no integer which can display the characteristic of an identity element in division.

INVERSE UNDER DIVISION

Since the identity for division doesn't exist, the question of inverse under division does not arise.

Note

For any integer a, $a \div a = \frac{a}{a} = a \times \frac{1}{a} = 1$.

HOTS

An ant is trying to climb a wall of height 45 m. For every move that it makes, it climbs a distance of 4 m and slips down by 1 m. In how many moves will it be able to climb the wall?

SOME EXAMPLES

Example 1: Verify whether the following statements are true or false:

(i) $-5 \div (-5) = 1$

(ii) $6 \div (-6) = 1$

(iii) $0 \div 6$ is undefined

(iv) $(-55) \div 0 = 0$

(v) $16 \div (-13) = (-13) \div 16$

(vi) $(-1) \div (-5) = \frac{1}{5}$

(vii) $-65 \div 1 = -65$

(viii) $51 \div (-17) = 3$

Solution: (i) We know that $(-a) \div (-b) = a \div b$

Thus, $-5 \div (-5) = 5 \div 5 = 1$, therefore the statement is true.

(ii) We know that $a \div (-b) = (-a) \div b = -(a \div b)$

Thus, $6 \div (-6) = -(6 \div 6) = -1$, therefore the statement is false.

(iii) As $0 \div 6 = 0$, the statement is false.

(iv) As $(-55) \div 0$ is not defined, the statement is false.

(v) We know that division is not commutative.

Thus, $16 \div (-13) \neq (-13) \div 16$, therefore the statement is false.

(vi) We know that $(-a) \div (-b) = a \div b$

Thus, $(-1) \div (-5) = 1 \div 5 = \frac{1}{5}$, therefore the statement is true.

(vii) We know that $a \div (-b) = (-a) \div b = -(a \div b)$

Thus, $-65 \div 1 = -(65 \div 1) = -65$, therefore the statement is true.

(viii) LHS $= 51 \div (-17) = -(51 \div 17) = -3$ and RHS $= 3$, therefore the statement is false.

Example 2: A shopkeeper earns a profit of ₹1 by selling a pencil and incurs a loss of 20 paise by selling an eraser.

(i) On a particular day, he incurred an overall loss of ₹10. On this day, he sold 35 pencils. How many erasers did he sell on this day?

(ii) On the very next day, he earned a profit of ₹25. On this day, he sold 100 erasers. How many pencils did he sell?

Solution: Profit means positive earning and loss means negative earning.

(i) Profit earned by selling one pencil = ₹1.

Profit earned by selling 35 pencils = ₹(35 × 1) = ₹35.

Total loss incurred = –₹10.

Total loss = Profit earned by selling pencils + Loss incurred by selling erasers

Therefore, loss incurred by selling erasers = Total loss – Profit earned by selling pencils

= ₹(–10 – 35) = ₹(–45)

Loss incurred by selling one eraser = –20 paise.

Total loss incurred on selling erasers = –₹45 = –4500 paise

So, number of erasers sold = (–4500) ÷ (–20) = 225 erasers

(ii) Loss incurred by selling 1 eraser = –20 paise.

∴ Loss incurred by selling 100 eraser = 100 × (–20) = –2000 paise = –₹20

Total profit = Profit earned by selling pencils + Loss incurred by selling erasers

∴ Profit earned by selling pencils = Total profit – Loss incurred by selling erasers

= ₹25 – (–₹20) = ₹45

So, number of pencils sold = 45 ÷ Profit on one pencil

= 45 ÷ 1 = 45.

Example 3: In a test containing 50 objective type questions, a student will be awarded +2 marks for every correct answer, –2 for every incorrect answer and 0 for not writing any answer. Mention the ways of scoring 94 marks in the test by a student.

Solution: Marks scored = +94

So, minimum correct answers = +94 ÷ (+2) = 47

Case 1

Correct answers = 47

Marks for one correct answers = +2

Marks for 47 correct answers = (47 × 2) = +94

Marks scored = +94

Marks obtained for incorrect answer = 0.

So, there is no incorrect answer.

Thus, 47 answers are correct and three questions are not attempted.

Case 2

Correct answers = 48.

Marks for 48 correct responses = (48 × 2) = +96.

Marks scored = +94

Marks obtained for incorrect answers = 94 – (+96) = –2.

Marks for one incorrect answer = –2.

Number of incorrect answers = (–2) ÷ (–2) = 1.

So, 48 answers are correct, one is incorrect and one question is not attempted.

Case 3

Correct answers = 49.

Marks from 49 correct answers = (49 × 2) = +98.

Marks scored = +94.

Marks obtained for incorrect answers = +94 – (+98) = –4.

Number of incorrect responses = (–4) ÷ (–2) = 2.

This gives the total number of questions = 49 + 2 = 51, but in actual the total number of questions is 50. So, this case is not possible.

So, the possible ways are:

47 correct answers, zero incorrect answer, three questions not attempted, and

48 correct answers, one incorrect answer, one question not attempted.

PRACTICE EXERCISE 1.4

(1) Evaluate each of the following division:

(i) (–50) ÷ 10 (ii) (–39) ÷ 13 (iii) 169 ÷ (–13) (iv) 245 ÷ (–1)

(v) (–34) ÷ (–34) (vi) 0 ÷ (–26) (vii) 6625 ÷ (–125) (viii) (–4352) ÷ (–17)

(2) Fill in the blanks:

(i) 54 ÷ ____ = 9 (ii) –72 ÷ ____ = 3 (iii) _____ ÷ (–5) = 7

(iv) 5 ÷ ____ = –1 (v) –5 ÷ _____ = –5 (vi) 1 ÷ _____ = –1

(vii) ____ ÷ (–675) = 0 (viii) ____ ÷ 198 = 2 (ix) ____ ÷ 254 = –7

(x) 1071 ÷ _____ = (–51)

(3) Evaluate the following:

(i) 42 ÷ {(–1) + (–2)} (ii) {84 ÷ (–2)} ÷ 7 (iii) (–44) ÷ {(–42) + (–2)}

(iv) {(–33) + (33)} ÷ 23 (v) {(–15) + 3} ÷ {(–7) + 4} (vi) (289 × 25 × 0) ÷ (–16)

(vii) {(–119) + 4} ÷ 23 (viii) {(–54) ÷ (–2)} ÷ 3 (ix) (24 ÷ 6) ÷ {(–6) ÷ 6}

(4) Simplify the following:

(i) 42 – 162 ÷ 3 × 2 (ii) (–34) × (–1) + (–117) ÷ (–9) (iii) 45 + 21 ÷ 7 – 6 × 5

(iv) (–3) + (–15) ÷ (–5) – 4 × (–4) (v) (–13) × (–7) – 22 ÷ 11

(5) Write five pairs of integers (a, b) such that $a \div b = -5$ [for example, (–25, 5)].

(6) Verify that, $a \div (b + c) \neq (a \div b) + (a \div c)$ for each of the following values of a, b and c:

(i) $a = -36$, $b = 4$ and $c = 2$ (ii) $a = -54$, $b = -3$ and $c = -6$

(iii) $a = -30$, $b = -5$ and $c = 3$

(7) The product of two integers is 240. If one integer is – 15, find the other integer.

(8) In Shillong, the temperature recorded for six days is as follows:

January 15	January 16	January 17	January 18	January 19	January 20
–5°C	–6°C	0°C	5°C	13°C	11°C

Find the average temperature for these six days.

$$\left[\text{Hint: Average temperature} = \frac{\text{Sum of temperatures on all six days}}{6}\right]$$

(9) A submarine descends from the surface of the water at the rate of 4m/s. At this rate, how long does it take for the submarine to reach –248 m from water level?

(10) The temperature of a rod was 10°C above zero. It was kept in a freezer at 2 p.m. If its temperature decreased at the rate of 3°C per minute, at what time would the temperature be 8°C below zero? And what would be the temperature at 2:15 p.m.?

(11) A multistorey building has 15 floors above the ground level each of height 9 m. It also has two floors in the basement each of height 6 m. A lift in the building moves at the rate of 2 m/s. If a man starts from 90 m above the ground, how long does it take for him to reach at first floor of the basement?

(12) In a class test, +5 marks are given for every correct answer and –3 marks are given for every incorrect answer and no marks for not attempting any question. Find the following:

(i) Arihant has scored 46 marks. If he has given 11 correct answers, then how many incorrect answers has he given?

(ii) Arihant has scored –7 marks in his test, though he has given four correct answers. How many questions has he answered incorrectly?

MULTIPLE CHOICE QUESTIONS

(1) On the following number line, (–5 × –2) is shown by the point:

(a) D. (b) F. (c) E. (d) G.

(2) A tanker contains 987 L of water. A tap fitted with a tanker starts leaking, and the water starts decreasing at the rate of 15 L per hour. How much water is left in the tanker after 14 hours?

(a) 777 L (b) 210 L (c) 1197 L (d) 787 L

(3) If *a* is a non-zero integer such that dividing *a* by –9 gives 18, then what should be the value of *a*?

(a) 162 (b) 2 (c) –162 (d) 2

(4) Match the integers in the following table such that their product lies between –20 and –8. Choose the correct option.

(i) –6	(ii) 9	(iii) –11	(iv) 5
(1) –1	(2) 1	(3) –2	(4) 3

(a) (i)-(4), (ii)-(1), (iii)-(2), (iv)-(3)
(b) (i)-(3), (ii)-(2), (iii)-(1), (iv)-(4)
(c) (i)-(4), (ii)-(3), (iii)-(2), (iv)-(1)
(d) (i)-(1), (ii)-(4), (iii)-(3), (iv)-(2)

COMPREHENSIVE EXERCISE

(1) If * is an operation such that for integers *a* and *b*, we have $a * b = a \times b + (a \times a - b \times b)$, then find

(i) (–4) * (–7) (ii) 5 * (–2) (iii) (–8) * 9

(2) **Given x, y, z and u are different integers such that $x = -9$ and $y = 1$. Using the following equations, find the values of z and u.**

(i) $x \times z = x$ (ii) $x + y = u$

(3) **Solve the following:**

(i) $61 + (-4) + 8 + (-12)$ (ii) 165×30 (iii) $14 \times (-7) \times (-16) \times (-1)$

(iv) $2128 \div (-152)$ (v) $(-304 + 10) \div (-14)$*

* For more practice questions refer to practice book.

CHAPTER CHECK-UP

- Integers are a collection of numbers, consisting of whole numbers and negative numbers.
- If a and b are integers, then $a + b$ and $a - b$ are also integers. We call it closure property.
- Addition is commutative for integers, that is, $a + b = b + a$ for all integers a and b.
- Addition is associative for integers, that is, $(a + b) + c = a + (b + c)$ for all integers a, b and c.
- Integer 0 is the identity under addition, that is, $a + 0 = 0 + a = a$ for every integer a.
- The commutative and associative properties are not true for subtraction of integers.
- For multiplication: for any two positive integers a and b, $(+a) \times (+b) = + ab$, $(-a) \times (-b) = (+ab)$, $(+a) \times (-b) = (-ab)$, $(-a) \times (+b) = (-ab)$.
- Product of even number of negative integers is positive, whereas the product of odd number of negative integers is negative.
- If a and b are integers, then $a \times b$ is also an integer. This is closure property for multiplication.
- Multiplication is commutative for integers, that is, $a \times b = b \times a$ for any two integers a and b.
- Integer 1 is the identity under multiplication, i.e. $1 \times a = a \times 1 = a$ for any integer a.
- Multiplication is associative for integers, that is, $(a \times b) \times c = a \times (b \times c)$ for any three integers a, b and c.
- Multiplication distributes over addition for integers, that is, for any three integers a, b and c, $a \times (b + c) = a \times b + a \times c$. This is called distributive property of multiplication over addition.
- Multiplication distributes over subtraction for integers, that is, for any three integers a, b and c, $a \times (b - c) = a \times b - a \times c$. This is called distributive property of multiplication over subtraction.
- For division: for any two positive integers a and b, $(+a) \div (+b) = + (a \div b)$, $(-a) \div (-b) = + (a \div b)$, $(+a) \div (-b) = -(a \div b)$, $(-a) \div (+b) = -(a \div b)$.
- The closure, commutative and associative properties are not true for division of integers.
- For any integer a, $a \div a = 1$, $a \div 1 = a$, $0 \div a = 0$ and $a \div 0$ is meaningless or undefined.

WEBLINKS:

http://www.mrmaisonet.com/index.php?/Integer-Quizzes/Integer-Division.html

Chapter 2

Rational Numbers

We know that for counting things, *natural numbers* (e.g. 1, 2, 3, …, etc.) are used. All the natural numbers, including zero, are called *whole numbers*. We also know that for every natural number, there exists an additive inverse, such as,–1, –2, –3 and –4, …, and these numbers, including whole numbers, form a collection of integers.

We also know that the sum, difference or product of two integers is always an integer; but the division of two integers may not always give an integer. For example, when 11 is divided by 3, the result is a fractional number. Also, we know that fractional numbers exist in the form $\frac{a}{b}$, where a and b are whole numbers, where b is not equal to 0.

We also know the opposite situations that involve numbers, such as, 250 m above sea level and 250 m below sea level are denoted by +250 m and –250 m, respectively. This can also be represented as $+\frac{1}{4}$ km and $\frac{-1}{4}$ km. Now, $\frac{-1}{4}$ is neither a fractional number nor an integer. Thus, to represent such numbers, we need to extend our number system.

RATIONAL NUMBERS

A rational number can be defined as the ratio of two integers expressed in the form $\frac{p}{q}$, where q is not equal to 0. Here, p is the numerator and q is the denominator. So, both $\frac{-7}{11}$ and $\frac{-13}{21}$ are examples of rational numbers.

OBSERVATION ABOUT RATIONAL NUMBERS

We can write 1 as $\frac{1}{1}$, 2 as $\frac{2}{1}$, 3 as $\frac{3}{1}$ and so on. We know 1, 2, 3,… are all natural numbers and can be expressed in $\frac{p}{q}$ form, which is a quotient of two integers.

Thus, we can say that all natural numbers are rational numbers, but all rational numbers such as $\frac{5}{7}$ need not be natural numbers.

Note that 0 can be expressed as $\frac{0}{1}, \frac{0}{2}, \frac{0}{-1}, \frac{0}{-2}$, ... and so on. In other words, $0 = \frac{0}{q}$, where q is any non-zero integer.

Thus, 0 can be expressed in $\frac{p}{q}$ form, where p is zero and q is any non-zero integer.

Thus, 0 is a rational number.

We have already discussed that 1 can be written as $\frac{1}{1}$, 2 can be written as $\frac{2}{1}$, 3 can be written as $\frac{3}{1}$ and so on.

Similarly, we can write –1 as $\frac{-1}{1}$, –2 as $\frac{-2}{1}$, –3 as $\frac{-3}{1}$ and so on.

And we know …, –3, –2, –1, 0, 1, 2, 3, … are all integers and can be expressed in $\frac{p}{q}$ form.

Thus, we can say all integers are rational numbers, but all rational numbers such as $\frac{5}{7}$, and $\frac{-9}{13}$ need not be integers.

The numbers such as $\frac{-4}{9}, \frac{9}{-13}, \ldots$ are all rational numbers, but they are not fractions since fractions have whole numbers in the numerator and denominator (the denominator not being 0). But all fractions such as $\frac{3}{7}, \frac{9}{13}, 1\frac{3}{5}, \ldots$ satisfy the definition of rational numbers.

Thus, we conclude that all fractions are rational numbers, but all rational numbers need not be fractions.

The entire decimal numbers, such as, 0.5, 4.6, 0.675, etc., can be written as fractions, for example,

$0.7 = \frac{7}{10}, 4.9 = \frac{49}{10}, 0.673 = \frac{673}{1000}$

Thus, all decimal numbers are fractions and hence are rational numbers.

Note

A rational number remains unchanged whenever its numerator and denominator are multiplied or divided by same non-zero integer.

So, we can say rational numbers include fractional and decimal numbers.

EQUIVALENT RATIONAL NUMBERS

We already know the method to find equivalent fractions and can also obtain equivalent rational numbers by the same method (by multiplying or dividing the numerator and the denominator of a rational number by the same non-zero integer).

Example:

$$\frac{-2}{5} = \frac{-2 \times 2}{5 \times 2} = \frac{-4}{10}$$

$$\frac{-2}{5} = \frac{-2 \times (-3)}{5 \times (-3)} = \frac{6}{-15}$$

$$\frac{-2}{5} = \frac{-2 \times 4}{5 \times 4} = \frac{-8}{20}$$

By multiplication

Similarly,

$$\frac{-25}{45} = \frac{-25 \div 5}{45 \div 5} = \frac{-5}{9}$$

$$\frac{34}{-36} = \frac{34 \div 2}{-36 \div 2} = \frac{17}{-18}$$

$$\frac{-44}{48} = \frac{-44 \div 2}{48 \div 2} = \frac{-44 \div 4}{48 \div 4}$$

By division

$$\therefore \quad \frac{-44}{48} = \frac{-22}{24} = \frac{-11}{12}$$

SOME EXAMPLES

Example 1: Find the value of x, if pairs of rational numbers below are equivalent.

(i) $\frac{6}{5}, \frac{x}{85}$ (ii) $-3\frac{1}{4}, \frac{x}{-24}$

Solution: (i) As we know that equivalent rational numbers have same value,

Thus, $\frac{6}{5} = \frac{x}{85}$

Since, 85 ÷ 5 = 17, hence 85 can be written as 5 × 17.

$$\frac{6}{5} = \frac{6 \times 17}{5 \times 17} = \frac{102}{85} = \frac{x}{85}$$

So, $x = 102$.

(ii) $-3\frac{1}{4}$ can be written as $\frac{-13}{4}$

We also know that equivalent rational numbers have the same value, thus

$$\frac{-13}{4} = \frac{x}{-24}$$

Since, $-24 \div 4 = -6$

$$\frac{-13}{4} = \frac{-13 \times (-6)}{4 \times (-6)} = \frac{78}{-24} = \frac{x}{-24}$$

So, $x = 78$.

Example 2: Express $\frac{2}{9}$ and $\frac{5}{24}$ as equivalent rational numbers having the same denominator.

Solution: First, we will find the common denominator for the rational numbers $\frac{2}{9}$ and $\frac{5}{24}$ by finding the LCM of 9 and 24, which is 72.

Now, 72 ÷ 9 = 8 and 72 ÷ 24 = 3.

Thus, $\frac{2}{9} = \frac{2 \times 8}{9 \times 8} = \frac{16}{72}$ and $\frac{5}{24} = \frac{5 \times 3}{24 \times 3} = \frac{15}{72}$.

Hence, the given rational numbers with the common denominator are $\frac{16}{72}$ and $\frac{15}{72}$.

Example 3: Give four rational numbers equivalent to rational number $\frac{4}{-13}$.

Solution: $\frac{4}{-13} = \frac{4 \times 2}{-13 \times 2} = \frac{4 \times 3}{-13 \times 3} = \frac{4 \times 4}{-13 \times 4} = \frac{4 \times 5}{-13 \times 5}$

Thus, $\frac{4}{-13} = \frac{8}{-26} = \frac{12}{-39} = \frac{16}{-52} = \frac{20}{-65}$.

POSITIVE AND NEGATIVE RATIONAL NUMBERS

A rational number $\frac{p}{q}$, where q not equal to 0, is said to be positive if both numerator and denominator are of the same sign, that is, either both are positive or both are negative.

For example, $\frac{8}{11}, \frac{-5}{-13}, \frac{9}{14}, \frac{-7}{-17}$, etc., are positive rational numbers.

A rational number $\frac{p}{q}$ is said to be negative if numerator and denominator are of opposite signs, that is, if numerator is positive, then denominator should be negative and vice-versa.

For example, $\frac{8}{-11}, \frac{-5}{13}, \frac{-9}{14}, \frac{-7}{17}$, etc., are negative rational numbers.

Note

The rational number zero is neither positive nor negative.

SOME EXAMPLES

Example 1: Write five negative rational numbers with denominator (i) – 13 and (ii) 56

Solution: (i) $\frac{5}{-13}, \frac{11}{-13}, \frac{9}{-13}, \frac{17}{-13}, \frac{21}{-13}$ [There could be more answers.]

(ii) $\frac{-19}{56}, \frac{-23}{56}, \frac{-71}{56}, \frac{-37}{56}, \frac{-43}{56}$ [There could be more answers.]

Example 2: Identify the following rational numbers as positive or negative rational numbers:

(i) $\frac{-7}{-11}$ (ii) $\frac{9}{10}$ (iii) $\frac{-4}{13}$ (iv) $\frac{6}{-17}$ (v) $\frac{3}{5}$

Solution: (i) $\frac{-7}{-11}$

Here, both numerator and denominator are negative. So, the rational number $\frac{-7}{-11}$ is positive.

(ii) $\frac{9}{10}$

Here, both numerator and denominator are positive. So, the rational number $\frac{9}{10}$ is positive.

(iii) $\frac{-4}{13}$

Here, numerator is negative and denominator is positive. So, the rational number $\frac{-4}{13}$ is negative.

(iv) $\frac{6}{-17}$

Here, numerator is positive and denominator is negative. So, the rational number $\frac{6}{-17}$ is negative.

(v) $\frac{3}{5}$

Here, both numerator and denominator are positive. So, the rational number $\frac{3}{5}$ is positive.

Example 3: (i) Express $\frac{-12}{17}$ as a rational number with the numerator –36.

(ii) Express $\frac{-12}{17}$ as a rational number with the denominator –102.

Solution: (i) Here, first we will find by what number should –12 be multiplied, so as to get –36 in the numerator. To obtain this, divide –36 by –12, after which we will get 3. So, we will multiply the numerator and the denominator by 3.

$$\frac{-12}{17} = \frac{-12 \times 3}{17 \times 3} = \frac{-36}{51}.$$

(ii) As done in the previous example, we have to first find by what number should 17 be multiplied, so as to get –102 in the denominator. To obtain this, divide –102 by 17, after which we will get – 6. So, we will multiply the numerator and the denominator by – 6.

$$\frac{-12}{17} = \frac{-12 \times (-6)}{17 \times (-6)} = \frac{72}{-102}.$$

STANDARD FORM OF RATIONAL NUMBERS

A rational number is said to be in a standard form if its denominator is positive and HCF of numerator and denominator is 1.

If a rational number is not in a standard form, then it can be reduced to the standard form. This can be done by the following few steps:

Step 1: First, obtain the rational number.

Step 2(a): If its denominator is positive, then divide its numerator and denominator by their HCF, ignoring the sign of numerator, whether positive or negative.

Step 2(b): If its denominator is negative, then divide its numerator and denominator by the negative of their HCF.

SOME EXAMPLES

Example 1: Reduce each of the following in standard form:

(i) $\frac{-81}{45}$ (ii) $\frac{36}{-28}$ (iii) $\frac{-15}{-10}$

Solution: (i) $\frac{-81}{45}$

As the denominator is positive, in order to express it in the standard form, we have to divide its numerator and denominator by the HCF of 81 and 45, that is 9.

45) 81 (1
−45
36) 45 (1
−36
9) 36 (4
−36
0

On dividing the numerator and the denominator of $\frac{-81}{45}$ by 9, we have

$$\frac{-81 \div 9}{45 \div 9} = \frac{-9}{5}.$$

Thus, the standard form of $\frac{-81}{45}$ is $\frac{-9}{5}$.

(ii) $\frac{36}{-28}$

As the denominator is negative, in order to express it in the standard form, we have to divide its numerator and denominator by the negative of the HCF of 36 and 28, that is 4.

28) 36 (1
−28
8) 28 (3
−24
4) 8 (2
−8
0

On dividing the numerator and the denominator of $\frac{36}{-28}$ by −4, we have

$$\frac{36 \div (-4)}{-28 \div (-4)} = \frac{-9}{7}.$$

Thus, the standard form of $\frac{36}{-28}$ is $\frac{-9}{7}$.

(iii) $\frac{-15}{-10}$

As the denominator is negative, in order to express it in the standard form, we have to divide its numerator and denominator by the negative of the HCF of 15 and 10, that is by −5.

10) 15 (1
−10
5) 10 (2
−10
0

On dividing the numerator and the denominator of $\frac{-15}{-10}$ by –5, we have $\frac{-15 \div (-5)}{-10 \div (-5)} = \frac{3}{2}$.

Thus, the standard form of $\frac{-15}{-10}$ is $\frac{3}{2}$.

PRACTICE EXERCISE 2.1

(1) Express each of the following integers as rational numbers:

(i) –6 (ii) 8 (iii) 0 (iv) –1

(2) Express $\frac{-8}{13}$ as a rational number with the following numerators:

(i) 16 (ii) 72 (iii) –88 (iv) 64 (v) 56

(3) Express $\frac{144}{-252}$ as a rational number with the following denominators:

(i) 21 (ii) –42 (iii) 14 (iv) 1260 (v) –21

(4) Express the following rational numbers in $\frac{p}{q}$ form:

(i) $4\frac{5}{6}$ (ii) $-3\frac{2}{5}$ (iii) $-7\frac{17}{19}$ (iv) $-13\frac{19}{23}$ (v) $8\frac{8}{9}$

(5) Write each of the following in standard form:

(i) $\frac{63}{49}$ (ii) $\frac{-6}{32}$ (iii) $\frac{209}{-152}$ (iv) $\frac{-245}{-325}$ (v) $\frac{-312}{663}$

(6) Which of the following are pairs of equivalent rational numbers?

(i) $\frac{4}{20}, \frac{1}{5}$ (ii) $\frac{3}{7}, \frac{7}{3}$ (iii) $\frac{-8}{13}, \frac{-32}{-52}$ (iv) $\frac{12}{7}, \frac{-60}{-35}$ (v) $\frac{-12}{21}, \frac{16}{28}$

(7) Which of the following rational numbers are equal?

(i) $\frac{32}{-40}, \frac{-4}{5}$ (ii) $\frac{-13}{9}, \frac{78}{-54}$ (iii) $\frac{8}{13}, \frac{54}{117}$ (iv) $\frac{14}{21}, \frac{2}{7}$

(8) Give three equivalent rational numbers for each of the following:

(i) $\frac{-7}{9}$ (ii) $\frac{12}{-17}$ (iii) $\frac{-8}{11}$ (iv) $\frac{9}{17}$

(9) Find an equivalent form of the rational numbers $\frac{5}{6}$ and $\frac{4}{30}$ that have the same denominator.

(10) Find an equivalent form of the rational numbers $\frac{4}{7}, \frac{8}{12}$ and $\frac{3}{28}$ that have the same denominator.

(11) For what value of x the pairs of the following rational numbers are equivalent?

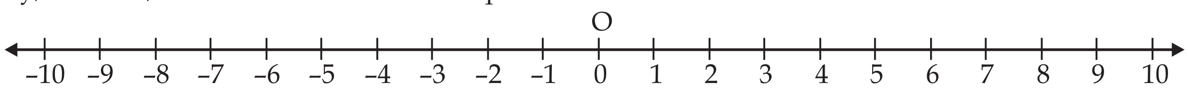
(i) $\frac{-6}{7}, \frac{x}{-42}$ (ii) $\frac{-8}{11}, \frac{x}{88}$ (iii) $\frac{8}{-5}, \frac{-24}{x}$ (iv) $\frac{3}{-10}, \frac{-27}{x}$

(12) Write the rational numbers whose numerator and denominator are respectively:

(i) 33 ÷ 3, 5 + 4 (ii) 3 × 4, (–36) ÷ 2

REPRESENTATION OF RATIONAL NUMBERS ON A NUMBER LINE

Earlier, we have learnt to represent integers on a number line. For that, we have to mark a point O to represent 0 on the number line. Then the points on the right and the left of 0 are marked at equal distances. The points marked on the right of 0 are positive and the points to the left of 0 are negative. From O, the pair 1 and –1 are equidistant. Similarly, 2 and –2, 3 and –3 and so on all are equidistant from O.

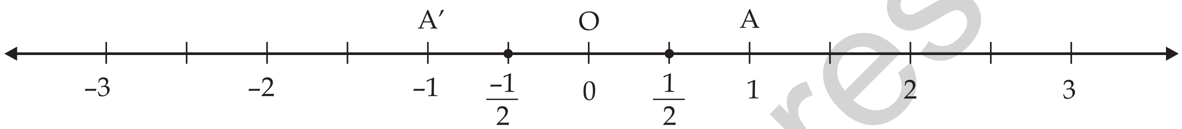


We have already studied the representation of fractions on a number line. Now, we will learn how to represent rational numbers on a number line.

First, draw a number line and mark a point O on it to represent the rational number 0. Now, if we mark a point A on the line to the right of O to represent 1, then OA = 1 unit. Similarly, if we choose point A′ on the left of O to represent –1, then OA′ = 1 unit.

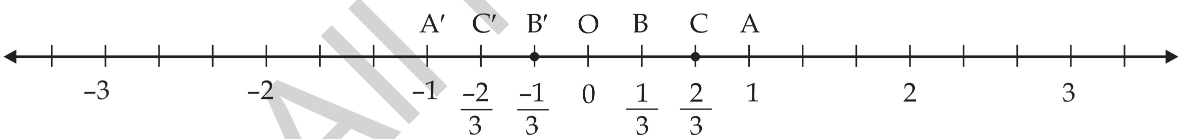
For rational numbers in standard form, if the denominator is greater than the numerator the rational number will always lie between 0 and 1 for positive rational numbers and 0 and –1 for negative rational numbers.

Let us now represent $\frac{1}{2}$ on a number line. To obtain this, we will have to divide OA into two equal parts, that is equal to the denominator of the rational number $\frac{1}{2}$ and to represent $-\frac{1}{2}$, OA′ is also divided into two equal parts.



Similarly, if we want to represent $\frac{2}{3}$ on the number line, we divide OA into three equal parts, that is equal to the denominator of the rational number $\frac{2}{3}$. Here first mark B will represent $\frac{1}{3}$ and the second mark C will represent $\frac{2}{3}$.

Now to represent $\frac{-2}{3}$, divide OA′ into three equal parts, equal to the denominator of the rational number $\frac{-2}{3}$. Here, also the first mark B′ will represent $\frac{-1}{3}$ and the second mark C′ will represent $\frac{-2}{3}$



A pair of rational numbers which are equidistant from O, but in opposite direction are called opposite pairs of rational numbers.

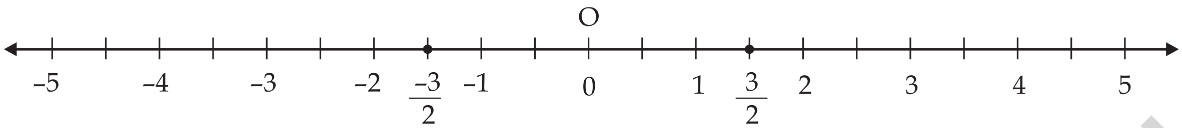
SOME EXAMPLES

Example 1: Represent $\frac{3}{2}$ and $-\frac{3}{2}$ on a number line.

Solution: First, we will draw the number line and mark the numbers... –3, –2, –1, 0, 1, 2,… at equal distances.

As $\frac{3}{2} = 1\frac{1}{2}$ lies halfway between 1 and 2. So, divide the distance between 1 and 2 into two equal parts. Similarly, $-\frac{3}{2} = -1\frac{1}{2}$ lies halfway between –1 and –2. So, divide the distance between –1 and –2 into two equal parts.

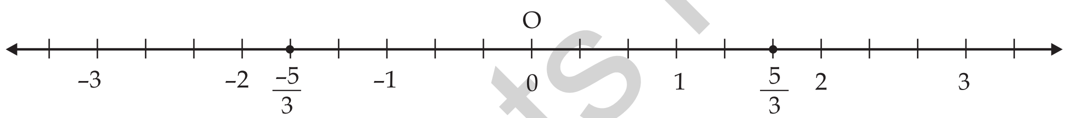
Mark $\frac{3}{2}$ at the midpoint of 1 and 2. Similarly, mark $-\frac{3}{2}$ at the midpoint of −1 and −2.



Example 2: Represent $\frac{5}{3}$ and $-\frac{5}{3}$ on a number line.

Solution: First, we will draw the number line and mark the numbers... −3, −2, −1, 0, 1, 2, 3,... at equal distances. As $\frac{5}{3} = 1\frac{2}{3}$ lies between 1 and 2, divide the distance between 1 and 2 into three (denominator of $\frac{5}{3}$) equal parts. Similarly, $-\frac{5}{3} = -1\frac{2}{3}$ lies between −1 and −2. So, divide the distance between −1 and −2 into three (denominator of $-\frac{5}{3}$) equal parts.

The numerator of fractional part of $1\frac{2}{3}$ is 2. So, the second mark between 1 and 2 will represent $\frac{5}{3} = 1\frac{2}{3}$. Similarly, the second mark between −1 and −2 will represent $-\frac{5}{3} = -1\frac{2}{3}$ as shown in the figure.



COMPARISON OF RATIONAL NUMBERS

In the previous classes, we have learnt how to compare integers and fractions. We already know that the number lying on the right of another number on a number line is always greater than the number and the number lying on the left of another number is smaller.

That is, every positive number is greater than 0 and every negative number is less than 0. Therefore, every positive number is greater than a negative number.

The same rules are valid for rational numbers also as mentioned below:

(1) All positive rational numbers are greater than 0.

(2) All negative rational numbers are smaller than 0.

(3) All positive rational numbers are greater than all negative rational numbers.

(4) All rational numbers represented by points on a number line are greater than all the rational numbers represented by points on their left and all rational numbers represented by points on a number line are smaller than all the rational numbers represented by points on their right.

We will now discuss the methods for comparing two rational numbers as follows:

Step 1: First, obtain the rational numbers.

Step 2: Write the rational number in standard form.

Step 3: Express each rational number to be compared, as equivalent rational numbers with common denominator.

Step 4: Now compare the numerators of rational numbers, the number having greater numerator is the greater rational number.

To compare two negative rational numbers, we can also compare them by ignoring their negative signs and then reverse the order. Let us look at some examples to understand this better.

SOME EXAMPLES

Example 1: Which of the two rational numbers is greater in each of the following?

(i) $\frac{1}{8}$ or 0 (ii) $\frac{8}{9}$ or $\frac{-7}{11}$ (iii) 0 or $\frac{9}{-17}$

Solution: (i) As $\frac{1}{8}$ is a positive rational number and every positive rational number is greater than 0.

Therefore, $\frac{1}{8} > 0$.

(ii) $\frac{8}{9}$ is a positive rational number and $\frac{-7}{11}$ is a negative rational number.

As every positive rational number is greater than negative rational number.

Therefore, $\frac{8}{9} > \frac{-7}{11}$.

(iii) As $\frac{9}{-17}$ is a negative rational number and every negative rational number is less than 0.

Therefore, $\frac{9}{-17} < 0$.

Example 2: Compare the following rational numbers:

(i) $\frac{3}{4}$ and $\frac{5}{6}$ (ii) $\frac{6}{-11}$ and $\frac{-5}{9}$

Solution: (i) $\frac{3}{4}$ and $\frac{5}{6}$

The LCM of 4 and 6 is 12.

Now, $\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$ and $\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$

Clearly, $9 < 10$

$\frac{9}{12} < \frac{10}{12}$

Thus, $\frac{3}{4} < \frac{5}{6}$.

(ii) $\frac{6}{-11}$ and $\frac{-5}{9}$

First, convert the negative denominator of $\frac{6}{-11}$ into positive by multiplying the numerator and denominator with –1.

$$\frac{6}{-11} = \frac{6 \times (-1)}{-11 \times (-1)} = \frac{-6}{11}$$

Let us now compare $\frac{-6}{11}$ and $\frac{-5}{9}$.

The LCM of the denominators 11 and 9 is 99.

$$\frac{-6}{11} = \frac{-6 \times 9}{11 \times 9} = \frac{-54}{99} \text{ and } \frac{-5}{9} = \frac{-5 \times 11}{9 \times 11} = \frac{-55}{99}$$

Clearly, $54 < 55, -54 > -55$

$$\frac{-54}{99} > \frac{-55}{99}$$

Hence, $\frac{-6}{11} > \frac{-5}{9}$

Thus, $\frac{6}{-11} > \frac{-5}{9}$

Example 3: Arrange $\frac{7}{12}, \frac{-5}{6}, \frac{11}{-36}, \frac{-17}{18}$ in:

(i) ascending order. (ii) descending order.

Solution: Note that $\frac{11}{-36}$ has a negative denominator. So, we multiply its numerator and denominator with –1.

$$\frac{11}{-36} = \frac{11 \times (-1)}{-36 \times (-1)} = \frac{-11}{36}$$

Let us now compare the rational numbers $\frac{7}{12}, \frac{-5}{6}, \frac{-11}{36}, \frac{-17}{18}$.

The LCM of denominators 12, 6, 36 and 18 is 36.

Now,

$$\frac{7}{12} = \frac{7 \times 3}{12 \times 3} = \frac{21}{36}$$

$$\frac{-5}{6} = \frac{-5 \times 6}{6 \times 6} = \frac{-30}{36}$$

$$\frac{-11}{36} = \frac{-11 \times 1}{36 \times 1} = \frac{-11}{36}$$

$$\frac{-17}{18} = \frac{-17 \times 2}{18 \times 2} = \frac{-34}{36}$$

(i) Ascending order

$$-34 < -30 < -11 < 21$$

$$\frac{-34}{36} < \frac{-30}{36} < \frac{-11}{36} < \frac{21}{36}$$

Hence, $\frac{-17}{18} < \frac{-5}{6} < \frac{-11}{36} < \frac{7}{12}$

Thus, the ascending order for the given numbers is $\frac{-17}{18}, \frac{-5}{6}, \frac{-11}{36}, \frac{7}{12}$.

(ii) Descending order

$$21 > -11 > -30 > -34$$

$$\frac{21}{36} > \frac{-11}{36} > \frac{-30}{36} > \frac{-34}{36}$$

Hence, $\frac{7}{12} > \frac{-11}{36} > \frac{-5}{6} > \frac{-17}{18}$

Thus, the descending order for the given numbers is $\frac{7}{12}, \frac{-11}{36}, \frac{-5}{6}, \frac{-17}{18}$.

Note

- Every rational number is either equal to, greater than or less than 0.
- If a, b and c are three rational numbers such that a is greater than b and b is greater than c, then a is greater than c.
- For any two rational numbers a and b, either a is greater than b or a is less than b or a is equal to b.

RATIONAL NUMBERS BETWEEN TWO RATIONAL NUMBERS

We know how to find the whole numbers between two whole numbers. For example, between 2 and 6, there are 3, 4 and 5 exactly three whole numbers. Similarly, between –4 and 2, there are –3, –2, –1, 0, 1 exactly five integers. There is no integer between –2 and –1, that is, between any two consecutive integers, there does not exist any integer. Thus, we can see that the number of integers between any two integers is limited or finite.

Let us now find the rational numbers between any two rational numbers, say $\frac{-6}{7}$ and $\frac{5}{7}$.
There are 10 integers between –6 and 5 those are: –5, –4, –3, –2, –1, 0, 1, 2, 3, 4.

Thus, we can say that the rational numbers between two rational numbers, $\frac{-6}{7}$ and $\frac{5}{7}$, are

$\frac{-5}{7}, \frac{-4}{7}, \frac{-3}{7}, \frac{-2}{7}, \frac{-1}{7}, \frac{0}{7}, \frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}$.

But the rational number $\frac{-6}{7}$ can be written as $\frac{-6 \times 2}{7 \times 2} = \frac{-12}{14}$ and $\frac{5}{7}$ as $\frac{5 \times 2}{7 \times 2} = \frac{10}{14}$.

Between –12 and 10, there are 21 integers: –11, –10, –9, –8, –7, –6, –5, –4, –3, –2, –1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Thus, we can get the rational numbers $\frac{-11}{14}, \frac{-10}{14}, \frac{-9}{14}, \frac{-8}{14}, \frac{-7}{14}, \frac{-6}{14}, \frac{-5}{14}, \frac{-4}{14}, \frac{-3}{14}, \frac{-2}{14}, \frac{-1}{14}, \frac{0}{14}, \frac{1}{14}, \frac{2}{14}, \frac{3}{14}, \frac{4}{14}, \frac{5}{14}, \frac{6}{14}, \frac{7}{14}, \frac{8}{14}, \frac{9}{14}$.

These rational number also lie between $\frac{-6}{7}$ and $\frac{5}{7}$.

On the similar lines, if we write $\frac{-6}{7} = \frac{-6 \times 3}{7 \times 3} = \frac{-18}{21}$ and $\frac{5}{7} = \frac{5 \times 3}{7 \times 3} = \frac{15}{21}$, we can find more rational numbers between $\frac{-6}{7}$ and $\frac{5}{7}$.

Thus, it would never end, and we will be able to find more rational numbers between $\frac{-6}{7}$ and $\frac{5}{7}$.

So, we can say there are infinite rational numbers between any two rational numbers.

SOME EXAMPLES

Example 1: List five rational numbers between –3 and –2.

Solution:

$$-3 = \frac{-3}{1} = \frac{(-3) \times 6}{1 \times 6} = \frac{-18}{6}$$

$$-2 = \frac{-2}{1} = \frac{(-2) \times 6}{1 \times 6} = \frac{-12}{6}$$

Multiply the numerator and the denominator with the number 1 more than the number of rational numbers required. Like here we need 5 rational numbers, so we have multiplied the given rational numbers by 6.

Since, $-18 < -12$,

we have, $\frac{-18}{6} < \frac{-12}{6}$.

Thus, $\frac{-17}{6}, \frac{-16}{6}, \frac{-15}{6}, \frac{-14}{6}, \frac{-13}{6}$ are the rational numbers between $\frac{-18}{6}$ and $\frac{-12}{6}$, that is, between –3 and –2.

Example 2: List seven rational numbers between $\frac{-3}{4}$ and $\frac{1}{-2}$.

Solution: Let us first write $\frac{1}{-2}$ in standard form $\frac{-1}{2}$. Now, we will make the denominators of $\frac{-3}{4}$ and $\frac{-1}{2}$ equal.The LCM of 4 and 2 is 4.

$$\frac{-3}{4} = \frac{-3\times 1}{4\times 1} = \frac{-3}{4} \text{ and } \frac{-1}{2} = \frac{-1\times 2}{2\times 2} = \frac{-2}{4}$$

$$\frac{-3\times 8}{4\times 8} = \frac{-24}{32} \text{ and } \frac{-2\times 8}{4\times 8} = \frac{-16}{32}$$

Multiply the numerator and the denominator with the number 1 more than the number of rational numbers required. Like here we need 7 rational numbers, so we have multiplied the given rational numbers by 8.

Since, $-24 < -16$

Thus, $\frac{-24}{32} < \frac{-16}{32}$

So, $\frac{-23}{32}, \frac{-22}{32}, \frac{-21}{32}, \frac{-20}{32}, \frac{-19}{32}, \frac{-18}{32}, \frac{-17}{32}$ are the rational numbers between $\frac{-24}{32}$ and $\frac{-16}{32}$, that is, between $\frac{-3}{4}$ and $\frac{-1}{2}$.

Note

To find rational numbers between any two rational numbers, the first-step is to make their denominators positive and equal. After making their denominators equal, multiply the numerator and the denominator with the number 1 more than the number of rational numbers to be inserted between two given rational numbers. Like here, we need seven rational numbers, so we will multiply it by 8.

PRACTICE EXERCISE 2.2

(1) Represent each of the following on a number line:

(i) $\frac{3}{5}, -\frac{3}{5}$ (ii) $\frac{-3}{7}, \frac{3}{7}$ (iii) $-1\frac{6}{9}, 1\frac{6}{9}$ (iv) $\frac{41}{5}, \frac{34}{5}, -\frac{4}{5}, \frac{21}{5}$

(2) Which of the two rational numbers is greater?

(i) $\frac{7}{8}$ or $\frac{8}{9}$ (ii) $\frac{-5}{7}$ or 0 (iii) $\frac{-6}{11}$ or $\frac{7}{-12}$ (iv) $\frac{-8}{13}$ or $\frac{5}{-9}$

(3) Arrange the following rational numbers in ascending order:

(i) $\frac{-3}{4}, \frac{-1}{3}, \frac{-13}{15}, \frac{2}{5}$ (ii) $\frac{-11}{16}, \frac{5}{-12}, \frac{-3}{8}, \frac{7}{-4}, \frac{13}{-20}$

(4) Arrange the following rational numbers in descending order:

(i) $\frac{-5}{3}, \frac{-7}{9}, \frac{-1}{6}, \frac{11}{27}$ (ii) $\frac{-8}{3}, \frac{-11}{6}, \frac{-3}{7}, \frac{-7}{6}$

(5) Write the five rational numbers between each of the following:

(i) – 4 and –5 (ii) $\frac{2}{7}$ and $\frac{1}{3}$ (iii) $\frac{-5}{6}$ and $\frac{1}{-4}$

OPERATIONS ON RATIONAL NUMBERS

Earlier we have learnt to add, subtract, multiply and divide integers and fractions. Now, we will learn basic operations on rational numbers.

ADDITION

RECALL

When we add the two integers having the same sign, we just add them as whole numbers and put the same sign. When one positive integer and one negative integer is added, we subtract them as whole numbers by considering the numbers without their sign and then put the sign of the bigger number with the subtraction obtained.

Let us add two rational numbers, say, $\frac{8}{5}$ and $\frac{-6}{5}$.

To add rational numbers having same denominator, simply add the numerator keeping the common denominator as it is.

So, $$\frac{8}{5} + \left(\frac{-6}{5}\right) = \frac{8 + (-6)}{5} = \frac{2}{5}$$

Similarly, if we want to add rational numbers, say, $\frac{-4}{7}$ and $\frac{2}{7}$.

We can add them as $\frac{-4}{7} + \frac{2}{7} = \frac{-4 + 2}{7} = \left(\frac{-2}{7}\right)$.

Now, to add the rational numbers with different denominators, first write the equivalent rational number with LCM as the common denominator and then simply add the numerators keeping the LCM as their denominator.

Example: Add $\frac{-3}{4}$ and $\frac{5}{6}$.

LCM of 4 and 6 is 12.

So, $\frac{-3}{4} = \frac{-3 \times 3}{4 \times 3} = \frac{-9}{12}$ and $\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$

Thus, $\frac{(-3)}{4} + \frac{5}{6} = \frac{(-9)}{12} + \frac{10}{12} = \frac{(-9)+10}{12} = \frac{1}{12}$.

ADDITIVE INVERSE

Let us find the sum of $\frac{-5}{11}$ and $\frac{5}{11}$.

$$\frac{-5}{11} + \frac{5}{11} = \frac{(-5)+5}{11} = \frac{0}{11} = 0.$$

Similarly, the sum of $\frac{4}{17}$ and $\frac{-4}{17}$ is $\frac{4+(-4)}{17} = \frac{0}{17} = 0$.

Therefore, $\frac{5}{11}$ is the additive inverse of $\frac{-5}{11}$ and vice-versa.

Similarly, $\frac{-4}{17}$ is the additive inverse of $\frac{4}{17}$ and vice-versa.

Thus, the additive inverse of a number is the number that when added to it, yields 0. This number is also known as ***opposite number***.

It is the number of the same magnitude but lies on the opposite of 0 on the number line.

SOME EXAMPLES

Example 1: Add $\frac{4}{18}, \frac{5}{21}$ and $\frac{-7}{24}$.

Solution: $\frac{4}{18} + \frac{5}{21} + \frac{(-7)}{24}$

The LCM of the denominators of the rational numbers, 18, 21 and 24 is 504.

$$\frac{4}{18} = \frac{4 \times 28}{18 \times 28} = \frac{112}{504}$$

$$\frac{5}{21} = \frac{5 \times 24}{21 \times 24} = \frac{120}{504}$$

$$\frac{(-7)}{24} = \frac{(-7) \times 21}{24 \times 21} = \frac{-147}{504}$$

$$\frac{4}{18} + \frac{5}{21} + \frac{(-7)}{24} = \frac{112}{504} + \frac{120}{504} + \frac{(-147)}{504} = \frac{112+120-147}{504} = \frac{85}{504}$$

Example 2: The level of the water mark above the danger level is $15\frac{3}{10}$ m; later it falls by $34\frac{3}{4}$ m. What is the level of water now?

Solution: The level of water above the danger mark is denoted by positive sign and the water level below the danger mark is denoted by negative sign.

Thus, the level of water from the initial mark will be $15\frac{3}{10}$ m.

Since, the water mark drops by $34\frac{3}{4}$ m.

So, new water level = $15\frac{3}{10} + \left(-34\frac{3}{4}\right) = \frac{153}{10} - \frac{139}{4}$

$= \frac{153 \times 2 - 139 \times 5}{20}$

LCM of 10 and 4 is 20.

$= \frac{306 - 695}{20} = \frac{-389}{20} = -19\frac{9}{20}$.

Since, it is negative, it means that the water level is $19\frac{9}{20}$ m below the danger mark.

SUBTRACTION

As we know, subtracting one integer from another is the same as adding the additive inverse of the second to the first.

Example: 13 – 21 = 13 + (–21).

We can use the similar method to subtract one rational number from another.

SOME EXAMPLES

Example 1: Subtract $\frac{-5}{7}$ from $\frac{6}{9}$.

Solution: We have to find out $\frac{6}{9} - \left(\frac{-5}{7}\right)$

$= \frac{6}{9}$ + Additive inverse of $\left(\frac{-5}{7}\right)$

$= \frac{6}{9} + \frac{5}{7}$

$= \frac{42 + 45}{63}$

LCM of 9 and 7 is 63.

$= \frac{87}{63} = \frac{29}{21}$

$= 1\frac{8}{21}$.

Note

So, we can conclude that to subtract one rational number from the other, we add the additive inverse of the rational number being subtracted to the other rational number.

Example 2: Subtract $\frac{-6}{7}$ from $\frac{-2}{9}$.

Solution: We have to find $\frac{-2}{9} - \frac{(-6)}{7} = \frac{-2}{9}$ + Additive inverse of $\frac{(-6)}{7}$

$= \frac{-2}{9} + \frac{6}{7}$

$$= \frac{-2 \times 7 + 6 \times 9}{63}$$

LCM of 9 and 7 is 63.

$$= \frac{-14 + 54}{63} = \frac{40}{63}.$$

Example 3: What should be added to $\frac{-6}{15}$ to get $\frac{-3}{20}$?

Solution: The given number is $\frac{-6}{15}$.

Sum of the given numbers = $\frac{-3}{20}$.

Therefore, the required number = $\frac{-3}{20} - \left(\frac{-6}{15}\right)$.

$$= \frac{-3}{20} + \text{Additive inverse of } \left(\frac{-6}{15}\right)$$

$$= \frac{-3}{20} + \frac{6}{15}$$

$$= \frac{-3 \times 3 + 6 \times 4}{60}$$ [LCM of 20 and 15 is 60]

$$= \frac{-9 + 24}{60}$$

$$= \frac{15}{60} = \frac{1}{4}.$$

PRACTICE EXERCISE 2.3

(1) Add the following rational numbers:

(i) $\frac{-6}{10}$ and $\frac{4}{6}$ (ii) $\frac{7}{8}$ and $\frac{4}{-12}$ (iii) $\frac{9}{13}$ and $\frac{-5}{14}$

(iv) $\frac{-3}{16}$ and $\frac{7}{24}$ (v) $\frac{5}{-18}$ and $\frac{4}{27}$

(2) Simplify the following:

(i) $\frac{1}{9} + \frac{-2}{7} + \frac{4}{14}$ (ii) $\frac{9}{11} + \frac{-5}{16} + \frac{3}{8}$ (iii) $\frac{-13}{8} + \frac{-8}{35} + \frac{3}{7}$ (iv) $\frac{4}{6} + \frac{-9}{24} + 1$

(3) Find the additive inverse of the following:

(i) 2 (ii) –8 (iii) $\frac{-67}{-45}$

(iv) $\frac{8}{-19}$ (v) $\frac{-25}{65}$ (vi) $\frac{15}{17}$

(4) Subtract the following:

(i) $\frac{8}{5}$ from $\frac{6}{25}$ (ii) $\frac{-12}{13}$ from $\frac{-15}{26}$ (iii) $\frac{-17}{23}$ from $\frac{19}{46}$

(iv) $\frac{-13}{24}$ from 2 (v) –8 from $\frac{17}{20}$

(5) Evaluate the following:

(i) $\frac{8}{13} - \frac{1}{5}$ (ii) $\frac{7}{10} - \frac{-5}{9}$ (iii) $\frac{-6}{7} - \frac{-12}{-25}$ (iv) $-6 - \frac{4}{7}$

(6) The sum of two rational numbers is $\frac{9}{-14}$. If one of them is $\frac{6}{12}$, find the other.

(7) What should be added to $\frac{-17}{-33}$ to get 0?

(8) What should be added to $\frac{-19}{39}$ to get $\frac{13}{38}$?

(9) Subtract

(i) the sum of $\frac{7}{5}$ and $\frac{-5}{7}$ from $\frac{9}{13}$. (ii) the sum of $\frac{8}{15}$ and $\frac{-9}{-30}$ from $\frac{12}{45}$.

(10) The age of Shreya is $3\frac{1}{7}$ less than her brother's age. If her brother's age is $5\frac{5}{6}$ years, find Shreya's age.

(11) If Aadi took $\frac{7}{9}$ hours to complete his project and his brother took $\frac{11}{12}$ hours, who has taken more time and by how much?

MULTIPLICATION

RECALL

If we multiply a positive integer and a negative integer, we will get a negative integer. However, the product of two negative integers is a positive integer, and the product of two positive integers is also a positive integer.

We can find the product of two rational numbers using the following steps:

Step 1: Multiply the numerators of two rational numbers.

Step 2: Multiply the denominators of two rational numbers.

Step 3: Product of two rational numbers = $\frac{\text{Product of their Numerators}}{\text{Product of their Denominators}}$.

Thus, we can say if $\frac{a}{b}$ and $\frac{c}{d}$ are any two rational numbers, then $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$.

Now, if we have to multiply a rational number by an integer, multiply the numerator by that integer, keeping denominator the same.

For example, multiply $\frac{-3}{4}$ by 5.

That is, $\frac{-3}{4} \times 5 = \frac{-15}{4}$.

MULTIPLICATIVE INVERSE OF A RATIONAL NUMBER

Let us find the product of $\frac{7}{19}$ and $\frac{19}{7}$.

Thus, $\frac{7}{19} \times \frac{19}{7} = 1$

Similarly, $\frac{-5}{9} \times \frac{-9}{5} = 1$

$\frac{7}{19}$ and $\frac{19}{7}$ are called multiplicative inverse of each other. Similarly, $\frac{-5}{9}$ and $\frac{-9}{5}$ are called multiplicative inverse of each other.

Thus, multiplicative inverse of a rational number is another rational number such that their product is 1. They are also called reciprocal of each other.

Note

The product of a rational number with its reciprocal is 1.

DIVISION

We know that division is the inverse of multiplication. So, to divide one rational number by the other rational number, we multiply one rational number (dividend) by the reciprocal of the other rational number (divisor).

Example: $\frac{8}{13} \div \frac{3}{5} = \frac{8}{13} \times (\text{reciprocal of } \frac{3}{5}) = \frac{8}{13} \times \frac{5}{3} = \frac{40}{39} = 1\frac{1}{39}$

Similarly, $-3\frac{2}{5} \div \frac{6}{7} = \frac{-17}{5} \times (\text{reciprocal of } \frac{6}{7}) = \frac{-17}{5} \times \frac{7}{6} = \frac{-119}{30} = -3\frac{29}{30}$

SOME EXAMPLES

Example 1: Simplify the following:

(i) $9 \times \frac{-4}{45}$ (ii) $\left(\frac{9}{36} \times \frac{-2}{7}\right) + \left(\frac{-3}{27} \times \frac{9}{7}\right) - \left(\frac{-3}{42} \times 7\right)$

Solution: (i) $9 \times \frac{-4}{45} = {}^{1}\cancel{9} \times \frac{-4}{\cancel{45}_5}$

$= \frac{-4}{5}$

Cancelling out the common factors

(ii) $\left(\frac{9}{36} \times \frac{-2}{7}\right) + \left(\frac{-3}{27} \times \frac{9}{7}\right) - \left(\frac{-3}{42} \times 7\right)$

$= \left(\frac{{}^{1}\cancel{9}}{\cancel{36}_{\cancel{4}2}} \times \frac{{}^{1}\cancel{-2}}{7}\right) + \left(\frac{{}^{1}\cancel{-3}}{\cancel{27}_{\cancel{9}1}} \times \frac{\cancel{9}}{7}\right) - \left(\frac{{}^{1}\cancel{-3}}{\cancel{42}_{\cancel{6}2}} \times \cancel{7}^{1}\right)$

$= \frac{-1}{14} + \left(\frac{-1}{7}\right) - \left(\frac{-1}{2}\right)$

$= \frac{(-1 \times 1) + (-1 \times 2) - (-1 \times 7)}{14}$ [LCM of 14, 7 and 2 is 14]

$$= \frac{-1-2+7}{14} = \frac{4}{14} = \frac{2}{7}$$

Example 2: Simplify the following:

(i) $\frac{4}{7} \times \frac{9}{5} \div \frac{8}{10}$ (ii) $\left(\frac{5}{7} \times 4\frac{1}{2} \div \frac{3}{28}\right) \div \frac{-1}{4}$

Solution: (i) $\frac{4}{7} \times \frac{9}{5} \times$ (Multiplicative inverse of $\frac{8}{10}$)

$$= \frac{4}{7} \times \frac{9}{5} \times \frac{10}{8}$$

$$= \frac{\cancel{4}^{1}}{7} \times \frac{9}{\cancel{5}_{1}} \times \frac{\cancel{10}^{\cancel{2}\,1}}{\cancel{8}_{\cancel{2}\,1}}$$

$$= \frac{1}{7} \times \frac{9}{1} \times \frac{1}{1} = \frac{9}{7}$$

(ii) $\left(\frac{5}{7} \times 4\frac{1}{2} \div \frac{3}{28}\right) \div \frac{-1}{4}$

$$= \left(\frac{5}{7} \times \frac{9}{2} \div \frac{3}{28}\right) \div \frac{-1}{4}$$

$$= \left(\frac{5}{7} \times \frac{9}{2} \times \text{Multiplicative inverse of } \frac{3}{28}\right) \div \frac{-1}{4}$$

$$= \left(\frac{5}{7} \times \frac{9}{2} \times \frac{28}{3}\right) \div \frac{-1}{4}$$

$$= \left(\frac{5}{\cancel{7}_{1}} \times \frac{\cancel{9}^{3}}{\cancel{2}_{1}} \times \frac{\cancel{28}^{\cancel{4}\,2}}{\cancel{3}_{1}}\right) \div \frac{-1}{4}$$

$$= \left(\frac{5}{1} \times \frac{3}{1} \times \frac{2}{1}\right) \times \left(\text{Multiplicative inverse of } \frac{-1}{4}\right)$$

$$= \left(\frac{5 \times 3 \times 2}{1 \times 1 \times 1}\right) \times \frac{-4}{1} = \frac{-120}{1} = -120$$

Example 3: A teacher has $\frac{3}{4}$ m of ribbon. She cuts it into pieces of $\frac{1}{28}$ m each. How many pieces she has cut?

Solution: Total length of the ribbon = $\frac{3}{4}$ m

Length of each piece = $\frac{1}{28}$ m

Number of strips cut = (Total length of the ribbon) $\div \frac{1}{28}$

$$= \frac{3}{4} \times \left(\text{Multiplicative inverse of } \frac{1}{28}\right)$$

$$= \frac{3}{4} \times 28 = 21 \text{ pieces}$$

PRACTICE EXERCISE 2.4

(1) Multiply the following:

(i) $\frac{4}{11}$ by $\frac{1}{13}$ (ii) $\frac{-19}{22}$ by $\frac{8}{-15}$ (iii) $\frac{7}{16}$ by $\frac{-3}{34}$ (iv) $\frac{8}{11}$ by $\frac{-5}{14}$ (v) $\frac{-18}{23}$ by 8

(2) Simplify the following:

(i) $\frac{5}{13}\times\frac{-9}{15}\times\frac{7}{18}$ (ii) $\left(\frac{-9}{17}+\frac{4}{34}\right)\times\frac{16}{25}$ (iii) $\frac{-13}{15}\times\frac{10}{26}+\frac{8}{14}\times\frac{-7}{16}$ (iv) $\frac{31}{4}\times\frac{-21}{20}\times\frac{17}{11}$

(3) Write the multiplicative inverse of the following rational numbers:

(i) $\frac{-16}{23}$ (ii) $\frac{51}{-42}$ (iii) 14 (iv) −13 (v) $\frac{-45}{-47}$ (vi) $\frac{7}{12}$

(4) Divide the following:

(i) $\frac{5}{6}$ by $\frac{25}{-36}$ (ii) $\frac{12}{23}$ by $\frac{-24}{11}$ (iii) $\frac{63}{8}$ by $\frac{7}{-16}$ (iv) $\frac{-108}{22}$ by $\frac{9}{88}$ (v) $\frac{-6}{9}$ by 3

(5) Simplify the following:

(i) $\left(\frac{6}{13}\div\frac{1}{7}\right)\times\frac{39}{28}$ (ii) $\left(\frac{-5}{12}\times\frac{36}{15}\right)\div\left(\frac{-6}{5}\times\frac{35}{18}\right)$ (iii) $\left(\frac{-9}{6}+\frac{5}{24}\right)\div\left(\frac{11}{4}-\frac{9}{16}\right)$

(6) Multiply:

(i) the sum of $\frac{1}{3}$ and $\frac{4}{7}$ by the sum of $\frac{2}{9}$ and $\frac{-3}{16}$.

(ii) the sum of $\frac{-3}{8}$ and $\frac{12}{17}$ by the product of $\frac{-2}{11}$ and $\frac{5}{22}$.

(7) Divide:

(i) the sum of $\frac{4}{7}$ and $\frac{-3}{8}$ by the difference of $\frac{9}{11}$ from $\frac{-2}{22}$.

(ii) the sum of $\frac{9}{-33}$ and $\frac{4}{22}$ by the product of $\frac{8}{13}$ and $\frac{-26}{3}$.

(8) The product of two rational numbers is $\frac{18}{4}$. If one number is $\frac{3}{-2}$, find the other.

(9) By what rational number should $\frac{-19}{42}$ be multiplied to get $\frac{95}{-7}$?

(10) By what rational number should $\frac{-21}{23}$ be divided to get $\frac{3}{46}$?

(11) The cost of $8\frac{2}{3}$ mL of oil is ₹$15\frac{3}{5}$. Find its cost per mL?

(12) Find the cost of $9\frac{1}{6}$ L of milk at the rate of ₹$23\frac{5}{11}$ per L.

HOTS

Sugandha walks $\frac{2}{5}$ km from a point P towards East and then from there $3\frac{7}{9}$ km towards West. What is her position now from a point P?

PROJECT WORK

Take the price of two seasonal fruits and two seasonal vegetables. Consider the price of vegetable as negative integer and the price of fruit as positive integer.

Now form two rational numbers by taking the price of fruit as numerator and the price of vegetable as denominator.

Now, add, subtract, mutiply and divide both the rational numbers.

MULTIPLE CHOICE QUESTIONS

(1) **What should be added to $\frac{13}{17}$ to get 1?**

(a) $\frac{3}{17}$ (b) $\frac{-4}{17}$ (c) $\frac{4}{17}$ (d) $\frac{-13}{17}$

(2) $\frac{17}{18} \times ? = \frac{34}{54}$

(a) $\frac{2}{3}$ (b) $\frac{-2}{3}$ (c) $\frac{3}{2}$ (d) $\frac{-3}{2}$

(3) $7\frac{2}{5} \div ? = -9\frac{13}{15}$

(a) $\frac{-3}{4}$ (b) $\frac{3}{4}$ (c) $\frac{4}{3}$ (d) $\frac{-4}{3}$

(4) **The product of the sum of $\frac{76}{18}$ and $\frac{11}{6}$ with the reciprocal of their difference is:**

(a) $2\frac{17}{43}$ (b) $1\frac{17}{43}$ (c) $2\frac{23}{43}$ (d) $2\frac{17}{43}$

(5) **The sum of two rational numbers is –6. If one of them is $\frac{21}{23}$, the other number is:**

(a) $-2\frac{5}{23}$ (b) $2\frac{5}{23}$ (c) $5\frac{2}{23}$ (d) $-6\frac{21}{23}$

COMPREHENSIVE EXERCISE

(1) **Write six rational numbers between**

(i) –9 and –8. (ii) $\frac{4}{9}$ and $\frac{1}{18}$. (iii) $\frac{-5}{7}$ and $\frac{3}{-14}$.

(2) **What should be added to $\left(\frac{6}{7} + \frac{-7}{6}\right)$ to get $\frac{9}{19}$?**

(3) **The product of two rational numbers is $\frac{57}{14}$. If one number is $\frac{3}{-7}$, find the other.**

(4) **A bucket has $\frac{3}{2}$ L of water in it. Rahul placed the bucket under a leaking tap so as to save water. He observed that $\frac{1}{3}$ L of water leaks from the tap in 1 hour. How much water will be there in the bucket after $2\frac{1}{2}$ hours?***

* For more practice questions refer to practice book.

CHAPTER CHECK-UP

- A rational number can be defined as a ratio of two integers expressed in $\frac{p}{q}$ form, where p and q are integers and q is not equal to 0. Here, p is the numerator and q is the denominator.
- All natural numbers are rational numbers, but all the rational numbers need not be natural numbers.
- All integers are rational numbers, but all the rational numbers need not be integers.
- All fractions are rational numbers, but all the rational numbers need not be fractions.
- All decimal numbers are fractions, and hence are rational numbers.
- A rational number $\frac{p}{q}$ is said to be positive, if both numerator and denominator have the same sign, that is, either both are positive or both are negative.
- A rational number $\frac{p}{q}$ is said to be negative, if both numerator and denominator have opposite signs, that is, if numerator is positive then denominator should be negative and vice-versa.
- The additive inverse of a number is the number that when added to it, yields 0. This number is also known as opposite number.
- To subtract one rational number from other, we add the additive inverse of the rational number being subtracted to the other rational number.
- Product of two rational numbers = $\frac{\text{Product of their numerators}}{\text{Product of their denominators}}$.
- Multiplicative inverse of a rational number is another rational number such that their product is one. They are also called reciprocal of each other.
- The product of a rational number with its reciprocal is 1.
- To divide one rational number by the other rational number, we multiply the rational number by the reciprocal of the other.

WEBLINKS:

http://www.basic-mathematics.com/rational-numbers.html

http://www.crctlessons.com/CRCT-Game-Numbers-and-Operations/crct-game.html

Chapter
3
Fractions

INTRODUCTION

Fraction is a part of a whole or a part of a collection. We have already studied fractions in our earlier grades. You may note that we deal with fractions in our daily life. The following are some examples:

(1) Rina went to a pizza party with her friends. She ate one fourth of a pizza.

(2) Shekhar drinks half a cup of milk with his breakfast.

(3) The tailor used one and a half metre cloth to stitch Riya's Jacket.

(4) The orange candy costs one fourth of a rupee.

Let us recall fractions, which we have already studied in our earlier grades.

RECALL

Proper fractions: A fraction is called a *proper fraction* if its numerator is less than its denominator.

A few examples of proper fractions are $\frac{1}{7}$, $\frac{2}{5}$ and $\frac{3}{8}$.

Improper fractions: A fraction is called an *improper fraction* if its numerator is greater than its denominator.

A few examples of improper fractions are $\frac{9}{8}$, $\frac{17}{10}$ and $\frac{24}{15}$.

Equivalent fractions: Every fraction has equivalent fractions. Equivalent fractions are obtained by multiplying or dividing the numerator and the denominator by the same number.

Some examples of equivalent fractions are $\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24} = \frac{20 \times 3}{24 \times 3} = \frac{60}{72}$.

Fractions in lowest terms: A fraction is said to be in its simplest form/lowest terms if the only common factor between the numerator and the denominator is 1. A fraction can be written in its simplest form by dividing its numerator and denominator by the HCF (highest common factor) of the numerator and the denominator.

ADDITION AND SUBTRACTION OF FRACTIONS

(1) Two like fractions can be added or subtracted by adding or subtracting the numerators and keeping the common denominator.

(2) Two unlike fractions are added or subtracted by first converting the fractions into equivalent fractions and then by adding or subtracting in the same manner as in the case of like fractions.

(3) To add or subtract mixed fractions, the whole parts are added or subtracted separately and the fractional parts are added or subtracted separately.

Now let us re-visit addition and subtraction of fractions with the help of the following examples:

SOME EXAMPLES

Example 1: Write the fraction equivalent to $\frac{27}{81}$ with numerator 3.

Solution: Equivalent fraction of $\frac{27}{81}$ with numerator 3 is

$$\frac{27 \div 9}{81 \div 9} = \frac{3}{9} \qquad [\text{as } 27 \div 9 = 3]$$

To make numerator, that is, 27 equal to 3, we have to divide the numerator by 9 and hence we divide the denominator also by 9.

Fraction $\frac{3}{9}$ is an equivalent fraction of $\frac{27}{81}$ with numerator 3.

Example 2: What fraction of 2 L is 400 mL?

Solution: We know that 1L = 1000 mL.

So, 2L = 2000 mL.

400 mL out of 2000 mL is $\frac{400}{2000} = \frac{400 \div 100}{20000 \div 100} = \frac{4}{20} = \frac{1}{5}$

Example 3: Shanaya made $3\frac{2}{7}$ L juice for the kids of the nearby slum. The kids drank $1\frac{2}{3}$ L juice. How much juice was left with Shanaya?

Solution: Amount of juice that Shanaya made = $3\frac{2}{7}$ L = $\frac{23}{7}$ L.

Amount of juice the kids drank = $1\frac{2}{3}$ L = $\frac{5}{3}$ L.

Amount of juice left with Shanaya = $3\frac{2}{7}$ L $- 1\frac{2}{3}$ L = $\frac{23}{7} - \frac{5}{3}$.

LCD = LCM of 7 and 3 = 7 × 3 = 21 (as 7 and 3 are co-prime numbers)

So, $\frac{23}{7} = \frac{23 \times 3}{7 \times 3} = \frac{69}{21}$ and

$\frac{5}{3} = \frac{5 \times 7}{3 \times 7} = \frac{35}{21}$

$\frac{69}{21} - \frac{35}{21} = \frac{69 - 35}{21} = \frac{34}{21}$

Note

LCD is the least common denominator.

We can write $\frac{34}{21}$ in mixed fraction $1\frac{13}{21}$.

So, $1\frac{13}{21}$ L juice is left with Shanaya.

Example 4: Kusum helps her mother by getting her vegetables from the market. She bought $5\frac{1}{4}$ kg onions and $2\frac{1}{3}$ kg potatoes. What is the total weight of vegetables that Kusum bought?

Solution: Weight of onion Kusum bought = $5\frac{1}{4}$ kg = $\frac{21}{4}$ kg.

Weight of potatoes Kusum bought = $2\frac{1}{3}$ kg = $\frac{7}{3}$ kg.

$\therefore$ Total weight = $5\frac{1}{4} + 2\frac{1}{3} = \frac{21}{4} + \frac{7}{3} = \frac{63 + 28}{12} = \frac{91}{12}$

(4 and 3 are co-prime numbers, hence LCM = $4 \times 3 = 12$, $21 \times 3 = 63$ and $7 \times 4 = 28$)

Fraction $\frac{91}{12}$ can be written in mixed fraction $7\frac{7}{12}$.

Example 5: Add $\frac{5}{2}$, $\frac{1}{5}$ and $\frac{4}{15}$

Solution: We have to find $\frac{5}{2} + \frac{1}{5} + \frac{4}{15}$

2	2,	5,	15
3	1,	5,	15
5	1,	5,	5
	1,	1,	1

LCM = $2 \times 3 \times 5 = 30$

The LCM of 2, 5 and 15 is 30.

So, as $2 \times 15 = 30$

$$\frac{5}{2} = \frac{5 \times 15}{2 \times 15} = \frac{75}{30}$$

As $5 \times 6 = 30$

Therefore, $\frac{1}{5} = \frac{1 \times 6}{5 \times 6} = \frac{6}{30}$

As $15 \times 2 = 30$

Therefore, $\frac{4}{15} = \frac{4 \times 2}{15 \times 2} = \frac{8}{30}$

So, $\frac{75}{30} + \frac{6}{30} + \frac{8}{30} = \frac{75 + 6 + 8}{30} = \frac{89}{30} = 2\frac{29}{30}$.

PRACTICE EXERCISE 3.1

(1) Solve and reduce the following to their lowest terms:

(i) $7 - \frac{6}{8}$ (ii) $9 + \frac{3}{5}$ (iii) $\frac{2}{6} + \frac{8}{5}$ (iv) $\frac{3}{8} - \frac{4}{15}$

(v) $\frac{5}{2} + \frac{1}{10} + \frac{6}{5}$ (vi) $7\frac{1}{3} - 4\frac{5}{9}$ (vii) $5\frac{1}{5} + 3\frac{7}{10}$

(2) Arrange the following:

(i) $\frac{4}{10}, \frac{2}{15}, \frac{7}{20}$ in ascending order (ii) $\frac{3}{5}, \frac{3}{10}, \frac{6}{15}$ in descending order

(3) Simplify:

(i) $\frac{5}{9} + \frac{1}{6} - \frac{2}{3}$ (ii) $13 - 7\frac{1}{3} - 5\frac{1}{6}$ (iii) $7\frac{1}{2} - \frac{1}{4} + 5\frac{5}{6}$

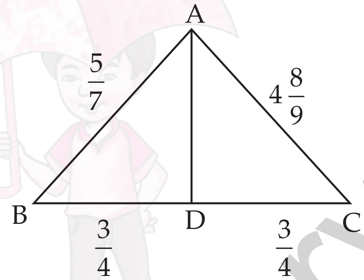
(4) What should be added to $5\frac{7}{9}$ to get 24?

(5) What should be subtracted from $6\frac{5}{12}$ to get $4\frac{4}{6}$?

(6) A magic show lasted for $2\frac{5}{6}$ hours. Out of this time, $1\frac{2}{3}$ hours was spent on arrangements. What was the actual duration of the show?

(7) A rectangular park is $7\frac{3}{5}$ m long and $7\frac{3}{10}$ m wide. Find its perimeter.

(8) In the adjoining figure, find the perimeter of triangle ABC.



(9) Of $\frac{6}{4}$ and $\frac{5}{16}$ which is greater and by how much?

(10) What should be subtracted from $\left[\frac{11}{18} + 3\frac{2}{6}\right]$ to get $\frac{31}{36}$?

(11) A 15 m long rope is cut into three pieces. If the length of two pieces is $4\frac{1}{3}$ m and $2\frac{2}{5}$ m, then find the length of the third piece.

MULTIPLICATION OF FRACTIONS

Suppose that the cost of 1 kg apples is ₹40. Find the cost of 3 kg apples.

By multiplication we can easily find the cost of 3 kg apples as $3 \times$ ₹40 = ₹120. So, the cost of 3 kg apples = ₹120.

Now, how will you find the answer if the cost of 1 kg apples is ₹ $25\frac{1}{2}$ and we have to find the cost of $2\frac{1}{4}$ kg apples?

Here, $25\frac{1}{2}$ has to be multiplied by $2\frac{1}{4}$, to get the answer. Both $25\frac{1}{2}$ and $2\frac{1}{4}$ are fractions. How will these be multiplied?

For multiplying such an amount, we should study multiplication of fractions.

MULTIPLICATION OF A WHOLE NUMBER BY A FRACTION

Let us do an activity. Cut out four squares from a white sheet of paper. Divide each of the squares into four equal parts with the help of your pencil and ruler as shown in the following images. Mark positions 1, 2, 3 and 4 on the small squares as shown:

1st square	
2	1
3	4

2nd square	
2	1
3	4

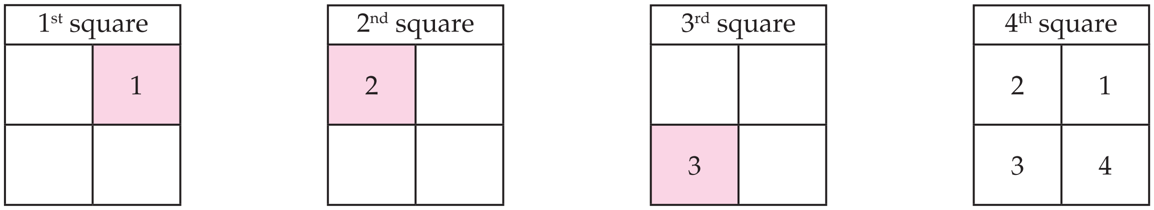
3rd square	
2	1
3	4

4th square	
2	1
3	4

Now, take three origami sheets of different colours, and cut them into squares of the same size as the biggest white square. Divide them into four parts, same as the white squares. Cut these squares to form four small squares of each colour.

Now place one coloured small square at position 1 of one of the 1st square.

Again, place another coloured square at position 2 of the 2nd square. Place the third coloured square at position 3 of the 3rd square as shown. The 4th square will remain as it is.

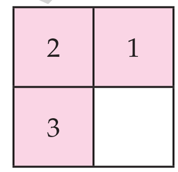


The coloured 1st square marked 1 denotes $\frac{1}{4}$.

The coloured 2nd square marked 2 denotes $\frac{1}{4}$.

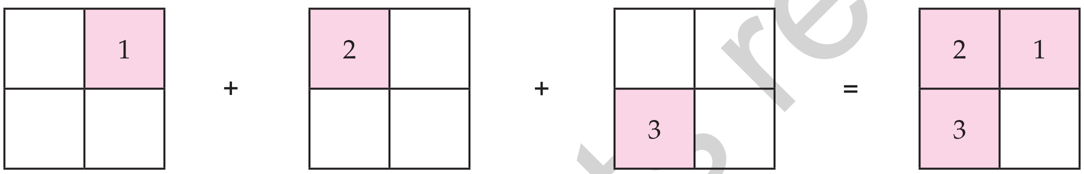
The coloured 3rd square marked 3 denotes $\frac{1}{4}$.

Now pick up these coloured squares and place them on the fourth square on their respective positions as shown.



Now, the coloured portion denotes $\frac{3}{4}$.

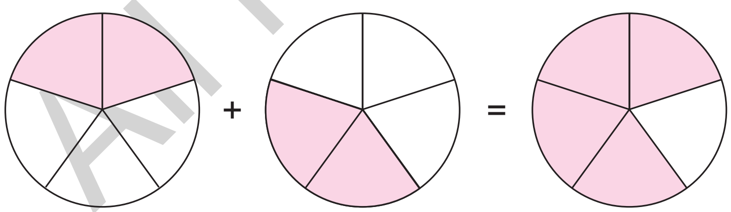
So, we can say that



$\frac{1}{4}$ $\frac{1}{4}$ $\frac{1}{4}$ $\frac{3}{4}$

That is, $3 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4}$ (Multiplication is nothing but repeated addition)

Similarly, $2 \times \frac{2}{5}$ can be represented as follows:



$\frac{2}{5}$ $\frac{2}{5}$ $\frac{4}{5}$

That is, $2 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} = \frac{4}{5}$

We can observe from the examples above that when we multiply a whole number by a fraction, the numerator is multiplied by the whole number and the denominator remains the same.

Thus,

$$5 \times \frac{1}{2} = \frac{5 \times 1}{2} = \frac{5}{2} = 2\frac{1}{2}$$

$$7 \times \frac{2}{12} = \frac{7 \times 2}{12} = \frac{14}{12} = \frac{7}{6} = 1\frac{1}{6}$$

$$2 \times \frac{2}{5} = \frac{2 \times 2}{5} = \frac{4}{5}$$

Note

- To multiply a whole number by a fraction, we multiply the whole number with the numerator of the fraction, keeping the denominator same. That is,

$$\text{Whole number} \times \frac{\text{Numerator}}{\text{Denominator}} = \frac{\text{Whole number} \times \text{Numerator}}{\text{Denominator}}$$

- To multiply a mixed fraction by a whole number, first convert the mixed fraction into an improper fraction and then multiply.

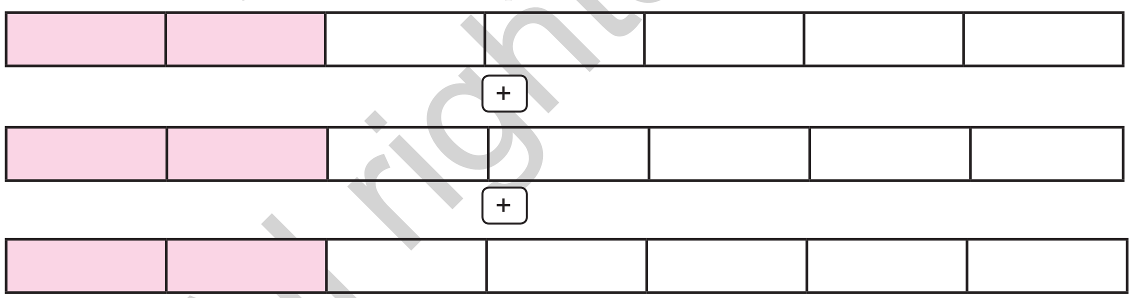
SOME EXAMPLES

Example 1: Show $3 \times \frac{2}{7} = \frac{6}{7}$ pictorially.

Solution: We have to show $3 \times \frac{2}{7} = \frac{6}{7}$ pictorially.

Let us take a rectangle divided into seven equal parts. Then we shall colour two parts out of the seven parts to show the fraction $\frac{2}{7}$.

This shows a rectangle divided into seven parts out of which two are coloured.



Now, count the total number of small coloured rectangles on each big rectangle and add. We get six small coloured rectangles. Now colour the six small rectangles out of the seven small rectangles as shown below.

$= \frac{6}{7}$

So, $3 \times \frac{2}{7} = \frac{2}{7} + \frac{2}{7} + \frac{2}{7} = \frac{2+2+2}{7} = \frac{6}{7}$

Example 2: Multiply the following, and reduce to the lowest terms:

(i) $5 \times \frac{3}{7}$ (ii) $12 \times \frac{2}{15}$ (iii) $3 \times 2\frac{1}{7}$

Solution: (i) $5 \times \frac{3}{7} = \frac{5 \times 3}{7} = \frac{15}{7} = 2\frac{1}{7}$

(ii) $12 \times \frac{2}{15} = \frac{12 \times 2}{15} = \frac{24}{15}$

Now, the HCF of 24 and 15 is 3; so both the numerator and the denominator are divided by 3.

$$\frac{24 \div 3}{15 \div 3} = \frac{8}{5}$$

Now, as $\frac{8}{5}$ is an improper fraction, it has to be converted into a mixed fraction.

$$\frac{8}{5} = 1\frac{3}{5}$$

(iii) $3 \times 2\frac{1}{7} = 3 \times \frac{15}{7} = \frac{3 \times 15}{7}$

$= \frac{45}{7} = 6\frac{3}{7}$

Note

- To multiply a whole number by a mixed fraction, convert the mixed fraction into an improper fraction.
- The product should be in the lowest terms and should be converted into a mixed fraction incase the product is an improper fraction.

Example 3: One cupcake requires $2\frac{2}{3}$ cups of flour and $1\frac{1}{3}$ cups of sugar. What will be the total quantity of each ingredient used in 10 such cupcakes?

Solution: Number of cups of flour required to make 1 cupcake = $2\frac{2}{3}$ cups = $\frac{8}{3}$ cups.

Number of cups of flour required to make 10 cupcakes = $10 \times 2\frac{2}{3}$

$= 10 \times \frac{8}{3} = \frac{80}{3} = 26\frac{2}{3}$ cups.

Number of cups of sugar required to make 1 cupcake = $1\frac{1}{3}$ cups = $\frac{4}{3}$ cups.

Number of cups of sugar required to make 10 cupcakes = $10 \times \frac{4}{3} = \frac{40}{3} = 13\frac{1}{3}$ cups. So, to make 10 cupcakes, $26\frac{2}{3}$ cups of flour and $13\frac{1}{3}$ cups of sugar are required.

FRACTION AS AN OPERATOR 'OF'

Look at the adjoining figure. Here, half of the box is filled with colour pencils. So, we can say that the filled portion is $\frac{1}{2}$ of 1.

Now look at two half-filled boxes. This shows that the filled portion is $\frac{1}{2}$ of 2.

Now, if we take three half-filled boxes, then the filled portion is $\frac{1}{2}$ of 3.

What happens if we combine the pencils of two boxes?

We get a complete pencil box, that is, $\frac{1}{2}$ of $2 = \frac{1}{2} \times 2 = 1$.

If we combine the pencils of three boxes, we have a complete box and a half filled box. That means $1 + \frac{1}{2} = \frac{1}{2}$ of $3 = \frac{1}{2} \times 3 = \frac{3}{2}$ So, we can see here that *'of'* means ***'multiplication'***.

SOME EXAMPLES

Example 1: Find the following:

(i) $\frac{1}{2}$ of 38°C (ii) $\frac{2}{3}$ of 180 m (iii) $\frac{2}{5}$ of 30 apples

Solution:

(i) $\frac{1}{2}$ of $38°\text{C} = \frac{1}{2} \times 38°\text{C} = \frac{38°\text{C}}{2} = 19°\text{C}$

(ii) $\frac{2}{3}$ of $180 \text{ m} = \frac{2}{3} \times 180 \text{ m} = 2 \times 60 \text{ m} = 120 \text{ m}$

(iii) $\frac{2}{5}$ of 30 apples $= \frac{2}{5} \times 30$ apples $= \frac{2 \times 30}{5} = \frac{60}{5} = 12$ apples

Example 2: A book has 350 pages. Rina read $\frac{1}{7}$ of the book in a day. How many pages has Rina read in a day?

Solution: Total number of pages in the book = 350.

Out of the total pages, Rina read $\frac{1}{7}$ of the total number of pages.

This means that Rina read $\frac{1}{7}$ of $350 = \frac{1}{7} \times 350 = \frac{350}{7} = 50$ pages

So, Rina read 50 pages in a day.

Example 3: In a class of 50 students, $\frac{1}{5}$ th of the total number of students like to play football and $\frac{2}{5}$ th of the total number of students like to play basketball and the remaining like to play cricket. What fractions of students like to play cricket?

Solution: Total number of students = 50.

Number of students who like to play football $= \frac{1}{5}$ of $50 = \frac{1}{5} \times 50 = \frac{50}{5} = 10$.

Number of students who like to play basketball $= \frac{2}{5}$ of $50 = \frac{2}{5} \times 50 = \frac{2 \times 50}{5} = \frac{100}{5} = 20$.

Number of students who play cricket = 50 – (10 + 20) = 50 – 30 = 20.

Fraction of students that play cricket $= \frac{20}{50} = \frac{2}{5}$.

Example 4: Cost of a taco is ₹ $20\frac{3}{4}$ and a sandwich is ₹ $15\frac{1}{2}$. Find the cost of 5 tacos and 9 sandwiches.

Solution: Cost of a taco = ₹ $20\frac{3}{4}$ = ₹ $\frac{83}{4}$

Cost of five tacos = $5 \times \frac{83}{4} = \frac{5 \times 83}{4} = \frac{415}{4} = ₹103\frac{3}{4}$.

Cost of a sandwich = $₹15\frac{1}{2} = ₹\frac{31}{2}$.

Cost of nine sandwiches = $9 \times \frac{31}{2} = \frac{9 \times 31}{2} = \frac{279}{2} = ₹139\frac{1}{2}$.

Hence, the cost of five tacos is $₹103\frac{3}{4}$ and the cost of nine sandwiches is $₹139\frac{1}{2}$.

HOTS

A water tank of a house contains 500 L water. Out of this, $\frac{1}{5}$ of the water is used in the kitchen and $\frac{2}{5}$ of the water is used for bathing and washing clothes. Remaining water is used for watering plants in the garden.

(1) How much water is used in the kitchen?

(2) How much water is used for bathing and washing clothes?

(3) What fraction of total quantity of water is used for watering in the garden?

MULTIPLICATION OF A FRACTION BY A FRACTION

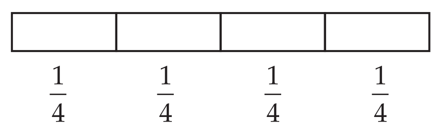
Let us multiply two fractions $\frac{1}{2}$ and $\frac{3}{4}$ pictorially.

We have to show $\frac{1}{2}$ of $\frac{3}{4}$.

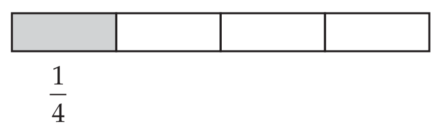
Step 1: Take a rectangular strip of any length and fold it into half $\left(\frac{1}{2}\right)$ twice.

That is, the strip will be divided into four parts.

So, here each section represents $\frac{1}{4}$ of the whole strip.

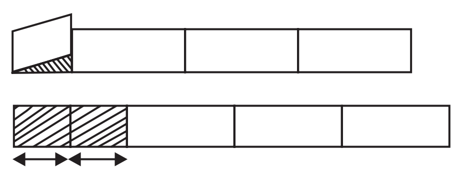


Step 2: Shade the first part of the strip. This shaded part represents $\frac{1}{4}$.

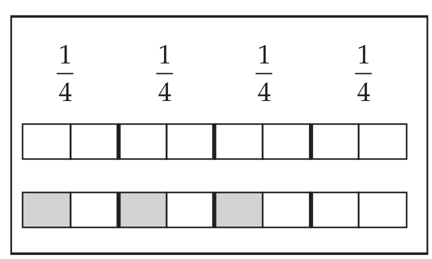


Step 3: Fold the shaded part into half. One half of the shaded part is $\frac{1}{2}$ of $\frac{1}{4}$.

That is, $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{2} \times \frac{1}{4} = \frac{1 \times 1}{2 \times 4} = \frac{1}{8}$



Step 4: Now divide each of the four parts into half. That means the strip is now divided into eight parts. Shade $\frac{1}{2}$ of the first three parts as shown.



Now, the shaded part is $\frac{1}{2}$ of $\frac{1}{4} + \frac{1}{2}$ of $\frac{1}{4} + \frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$

Now as we have shaded half of each of the first three parts, that is we have shaded

$$\frac{1}{2} \text{ of } \frac{3}{4} = \frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}$$

We observe here that when we multiply two fractions, the numerator of the first fraction is multiplied by the numerator of the second fraction and the denominator of the first fraction is multiplied by the denominator of the second fraction.

Note

Product of two fractions = $\frac{\text{Product of numerators}}{\text{Product of denominators}}$

We can also extend this rule to three fractions. Let us take the following example:

$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{5} = \frac{1 \times 2 \times 3}{2 \times 3 \times 5} = \frac{6}{30} = \frac{60 \div 6}{30 \div 6} = \frac{1}{5}$ (The HCF of 6 and 30 is 6, so we divide both the numerator and the denominator by 6.)

Alternatively,

We can also cancel the common numerators and denominators directly in this step.

$$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{5}$$

We can also cancel the common numerators and denominators directly in this step.

$$\frac{1}{\cancel{2}} \times \frac{\cancel{2}}{\cancel{3}} \times \frac{\cancel{3}}{5}$$

$$= \frac{1}{5}$$

$\frac{1}{②} \times \frac{②}{3} \times \frac{3}{5}$

$\frac{1}{1} \times \frac{1}{③} \times \frac{③}{5}$

$\frac{1}{1} \times \frac{1}{1} \times \frac{1}{5} = \frac{1}{5}$

SOME EXAMPLES

If mixed fractions are to be multiplied, first convert them into improper fraction and then multiply.

Example 1: Simplify: $2\frac{1}{3} \times 3\frac{1}{5}$.

Solution: $2\frac{1}{3} = \frac{7}{3}$ and $3\frac{1}{5} = \frac{16}{5}$

So, $2\frac{1}{3} \times 3\frac{1}{5} = \frac{7}{3} \times \frac{16}{5} = \frac{7 \times 16}{3 \times 5} = \frac{112}{15}$

Example 2: Evaluate:

(i) $5 \times \frac{1}{6} \times 2\frac{1}{3}$ (ii) $2 \times 3\frac{1}{5} \times 3\frac{2}{7}$ (iii) $5\frac{1}{7} \times 2\frac{1}{3} - 2\frac{2}{3} \times 3\frac{1}{2}$.

Solution:

(i) $5 \times \frac{1}{6} \times 2\frac{1}{3} = \frac{5 \times 1}{6} \times \frac{7}{3} = \frac{5}{6} \times \frac{7}{3} = \frac{5 \times 7}{6 \times 3} = \frac{35}{18}$

(ii) $2 \times 3\frac{1}{5} \times 3\frac{2}{7} = 2 \times \frac{16}{5} \times \frac{23}{7} = \frac{2 \times 16 \times 23}{5 \times 7} = \frac{736}{35}$

(iii) Using BODMAS, we do multiplication first.

Let us solve the two terms separately.

$$5\frac{1}{7} \times 2\frac{1}{3} = \frac{36}{7} \times \frac{7}{3} = \frac{36 \times 7}{7 \times 3} = \frac{252}{21} = 12$$

$$2\frac{2}{3} \times 3\frac{1}{2} = \frac{8}{3} \times \frac{7}{2} = \frac{56}{6}$$

As the HCF of 56 and 6 is 2, so we divide both the numerator and the denominator by 2.

$$\frac{56 \div 2}{6 \div 2} = \frac{28}{3}$$

Now let us subtract:

$$12 - \frac{28}{3} = \frac{12 \times 3}{3} - \frac{28}{3} = \frac{36}{3} - \frac{28}{3} = \frac{36 - 28}{3} = \frac{8}{3}$$

So, $5\frac{1}{7} \times 2\frac{1}{3} - 2\frac{2}{3} \times 3\frac{1}{2} = \frac{8}{3} = 2\frac{2}{3}$

Example 3: Rehan walks $\frac{2}{3}$ of a km in 1 hour. How much distance does he walk in $2\frac{2}{3}$ hours?

Solution: Distance walked by Rehan in 1 hour = $\frac{2}{3}$ of a km = $\frac{2}{3} \times 1$ km = $\frac{2}{3}$ km.

So, distance walked by Rehan in $2\frac{2}{3}$ hours = $\frac{2}{3}$ km $\times\ 2\frac{2}{3} = \frac{2}{3} \times \frac{8}{3}$ km $= \frac{2 \times 8}{3 \times 3}$ km

$= \frac{16}{9}$ km $= 1\frac{7}{9}$ km

Example 4: Of the total runs scored by Dravid in his career, he scored $\frac{3}{4}$ in the test matches. Shikhar made $\frac{2}{5}$ of the runs made by Dravid in test matches. What fraction of runs did Shikhar make?

Solution: Here, the fraction of runs made by Dravid in test matches = $\frac{3}{4}$

Shikhar made $\frac{2}{5}$ of the runs made by Dravid.

So, runs made by Shikhar = $\frac{2}{5}$ of $\frac{3}{4} = \frac{2}{5} \times \frac{3}{4} = \frac{2 \times 3}{5 \times 4} = \frac{\cancel{2} \times 3}{5 \times \cancel{4}_2}$

$= \frac{3}{5 \times 2} = \frac{3}{10}$

Cancelling out the common factors.

Example 5: The weight of an object on the moon is $\frac{1}{6}$ its weight on Earth. If an object weighs $4\frac{2}{5}$ kg on Earth,what will be the weight of the object on moon?

Solution: Weight of the object on Earth = $4\frac{2}{5}$ kg = $\frac{22}{5}$ kg.

Weight of the object on moon = $\frac{1}{6}$ of $\frac{22}{5} = \frac{1}{6} \times \frac{22}{5} = \frac{^{11}\cancel{22}}{_3\cancel{6} \times 5} = \frac{11}{3 \times 5} = \frac{11}{15}$.

Hence, the weight of the object on moon = $\frac{11}{15}$ kg.

Example 7: A student multiplied two mixed fractions in the following manner:

$3\frac{2}{3} \times 4\frac{3}{5} = 12\frac{2}{5}$. What is the error made by the student? Write the correct answer.

Solution: Error: To multiply two mixed fractions, the first step is to convert the mixed fractions into improper fractions and then multiply the numerators and the denominators.

Correct answer: $3\frac{2}{3} \times 4\frac{3}{5} = \frac{11}{3} \times \frac{23}{5} = \frac{11 \times 23}{3 \times 5} = \frac{253}{15} = 16\frac{13}{15}$

VALUE OF PRODUCTS

In the case of whole numbers, the product of two whole numbers is always greater than the two whole numbers. To do such kind of comparison in the case of fractions, let us do some investigations as mentioned in the following sections.

PRODUCT OF TWO PROPER FRACTIONS

The product of two proper fractions is less than both the fractions.

Let us check the product of two proper fractions by taking a few examples as in the following table:

Product	Comparison	Observation
(1) $\frac{2}{3} \times \frac{5}{6} = \frac{10}{18}$	$\frac{10}{18} = < \frac{2}{3}\left(\frac{2 \times 6}{3 \times 6} = \frac{12}{18}\right)$ $\frac{10}{18} = < \frac{5}{6}\left(\frac{5 \times 3}{6 \times 3} = \frac{15}{18}\right)$	The product is less than both the fractions.
(2) $\frac{1}{3} \times \frac{3}{7} = \frac{3}{21}$	$\frac{3}{21} < \frac{1}{3}\left(\frac{3}{21}, \frac{1 \times 7}{3 \times 7} = \frac{7}{21}\right)$ $\frac{3}{21} < \frac{3}{7}\left(\frac{3 \times 3}{7 \times 3} = \frac{9}{21}\right)$	The product is less than both the fractions.

PRODUCT OF TWO IMPROPER FRACTIONS

The product of two improper fractions is greater than both the fractions.

Let us check the product of two improper fractions by taking a few examples as in the following table:

Product	Comparison	Observation
(1) $\frac{7}{5} \times \frac{8}{3} = \frac{56}{15}$	$\frac{56}{15} > \frac{7}{5}\left(\frac{7 \times 3}{5 \times 3} = \frac{21}{15}\right)$ $\frac{56}{15} > \frac{8}{3}\left(\frac{8 \times 5}{3 \times 5} = \frac{40}{15}\right)$	The product is greater than both the fractions.
(2) $\frac{4}{3} \times \frac{8}{7} = \frac{32}{21}$	$\frac{32}{21} > \frac{4}{3}\left(\frac{4 \times 7}{3 \times 7} = \frac{28}{21}\right)$ $\frac{32}{21} > \frac{8}{7}\left(\frac{8 \times 3}{7 \times 3} = \frac{24}{21}\right)$	The product is greater than both the fractions.

PRODUCT OF A PROPER AND AN IMPROPER FRACTION

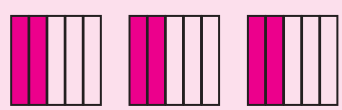
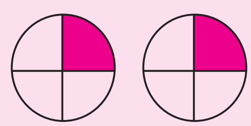
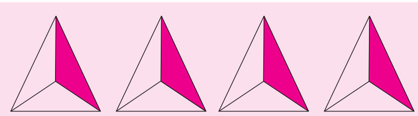
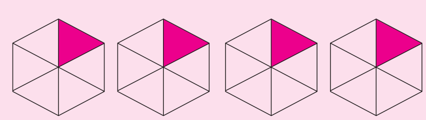
> The product of a proper fraction and an improper fraction is less than the improper fraction and greater than the proper fraction being multiplied.

Let us now check the product of a proper fraction and an improper fraction as in the following table:

Product	Comparison	Observation
(1) $\frac{6}{5}\times\frac{2}{7}=\frac{12}{35}$	$\frac{12}{35}<\frac{6}{5}\left(\frac{6\times7}{5\times7}=\frac{42}{35}\right)$ $\frac{12}{35}>\frac{2}{7}\left(\frac{2\times5}{7\times5}=\frac{10}{35}\right)$	Product is less than improper fraction and greater than proper fraction.
(2) $\frac{3}{4}\times\frac{11}{7}=\frac{33}{28}$	$\frac{33}{28}<\frac{11}{7}\left(\frac{11\times4}{7\times4}=\frac{44}{28}\right)$ $\frac{33}{28}>\frac{3}{4}\left(\frac{3\times7}{4\times7}=\frac{21}{28}\right)$	Product is less than improper fraction and greater than proper fraction.

PRACTICE EXERCISE 3.2

(1) Match the two columns in the following table:

Multiplication of fractions	Pictorial representation
(i) $2\times\frac{1}{4}$	
(ii) $3\times\frac{2}{5}$	
(iii) $4\times\frac{1}{6}$	
(iv) $4\times\frac{1}{3}$	

(2) Multiply and reduce to the lowest form, and then convert into mixed fraction:

(i) $8 \times \frac{5}{4}$ (ii) $9 \times \frac{17}{3}$ (iii) $7 \times \frac{1}{8}$ (iv) $24 \times \frac{7}{8}$

(v) $\frac{7}{4} \times 2$ (vi) $\frac{3}{8} \times 11$ (vii) $7\frac{4}{5} \times 1000$ (viii) $\frac{13}{3} \times 5$

(3) Find the product:

(i) $\frac{4}{9} \times \frac{3}{24}$ (ii) $\frac{2}{7} \times \frac{35}{16}$ (iii) $\frac{5}{11} \times \frac{77}{30}$ (iv) $\frac{9}{5} \times \frac{30}{27}$

(v) $\frac{9}{23} \times 7\frac{2}{3}$ (vi) $5\frac{9}{11} \times 9\frac{5}{8}$ (vii) $4\frac{3}{13} \times \frac{2}{5}$

(4) Simplify:

(i) $\frac{3}{5} \times \frac{7}{6} \times \frac{5}{14} \times \frac{2}{9}$ (ii) $\frac{8}{11} \times \frac{33}{2} \times \frac{7}{3} \times \frac{6}{21}$ (iii) $5\frac{2}{5} \times 3\frac{3}{7} \times 2\frac{2}{9} \times \frac{7}{4}$

(iv) $\frac{25}{11} \times \frac{16}{28} \times \frac{21}{20} \times \frac{88}{42}$ (v) $1\frac{3}{4} \times 2\frac{1}{7} + 3\frac{3}{5} \times \frac{4}{9}$ (vi) $4\frac{2}{5} \times 1\frac{4}{11} - 5\frac{2}{8} \times \frac{3}{7}$

(5) Find the answer:

(i) $\frac{1}{2}$ of 24 (ii) $\frac{5}{8}$ of 72 (iii) $\frac{7}{9}$ of 63

(iv) $\frac{2}{3}$ of 36 (v) $\frac{3}{5}$ of 45 (vi) $\frac{2}{7}$ of 7 kg

(vii) $\frac{5}{6}$ of an hour (in minutes) (viii) $\frac{1}{2}$ of ₹1 (in paise) (ix) $\frac{2}{3}$ of a year (in months)

(x) $\frac{2}{5}$ of 105° (xi) $\frac{2}{25}$ of a kilometer (in metres) (xii) $\frac{13}{17}$ of 51

(6) Find and reduce the answer to the lowest form, and then convert it into mixed fraction:

(i) $\frac{3}{7}$ of $2\frac{4}{5}$ (ii) $\frac{4}{9}$ of $4\frac{2}{4}$ (iii) $\frac{5}{8}$ of $1\frac{4}{5}$

(7) Suppose that kerosene oil is sold at ₹$15\frac{3}{7}$ per litre. Find the cost of $8\frac{1}{6}$ L of kerosene.

(8) Find the area of a square of side $6\frac{1}{2}$ m.

(9) Gautam earns ₹48,000 per month. He spends $\frac{5}{8}$ of his income and deposits rest of the money in a bank. How much money does he deposit in the bank each month?

(10) There are 81 passengers in a plane. Out of these, $\frac{4}{9}$ are males $\frac{2}{9}$ are females and rest are children. Find the number of children in the plane.

(11) Ria collected 87 game cards, whereas Nikhil collected $2\frac{1}{3}$ times more game cards than Ria. How many game cards did Nikhil collect?

(12) A wall $7\frac{1}{11}$ m high has to be built. On the first day, the mason built $\frac{2}{13}$ of the wall. What height of the wall is yet to be built?

(13) A total of 14 boards are stacked on the top of each other. The thickness of each board is $3\frac{2}{5}$ cm. How high is the stack?

HOTS

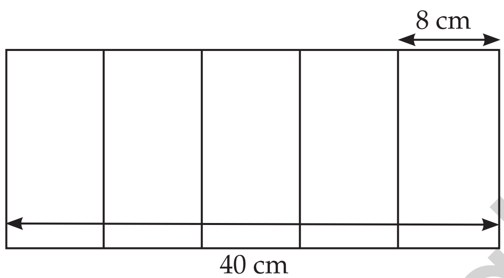
(1) Five students stand in a row such that the distance between two successive students is always $1\frac{1}{4}$ m. Find the total distance between the first and the last student.

(2) Both the opening players together scored $\frac{2}{3}$ of the total runs made by the team in a video game. However, Immanuel scored $\frac{1}{3}$ only of what opening players scored together. How many runs were scored by Immanuel alone if total score of the team was 333?

(3) Veronica reads a book for $\frac{3}{4}$ hours in a day. If she reads the entire book in 8 days. How many hours in all were required by her to read the book?

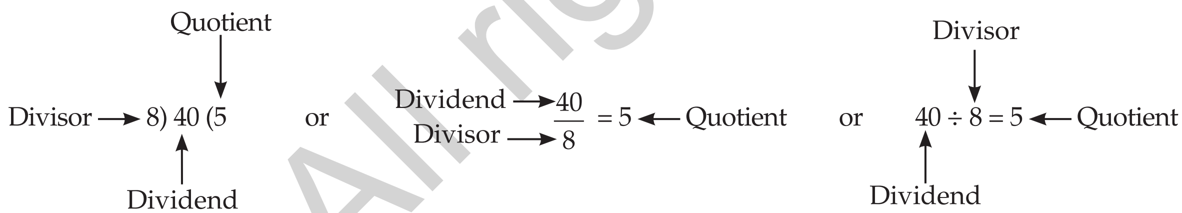
DIVISION OF FRACTIONS

A group of students were preparing for a social studies project. They marked a rectangle of 40 cm length and any width on a thermocol sheet. They wanted to know how many 8 cm sections could cover the 40 cm sheet. There can be three different ways to write this.



→ 8)40(or $\frac{40}{8}$ or 40 ÷ 8

The students could see that when 40 cm was divided into 8 cm sections, they obtained 5 such sections.



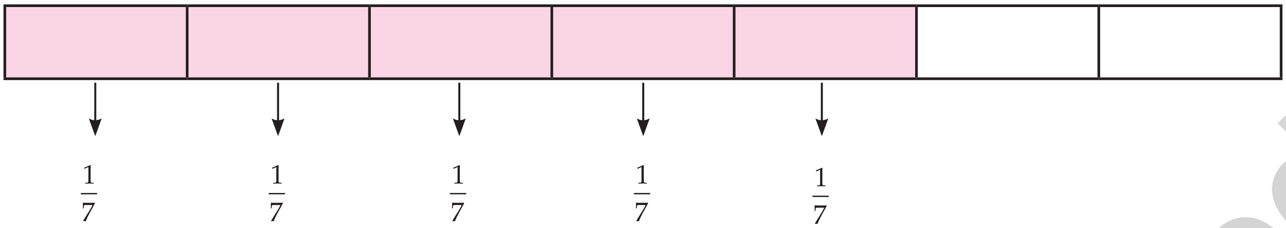
In each problem, 40 is ***dividend*** (the number that is being divided). The ***divisor*** is 8 (the number which divides). The answer to the problem, that is, 5 is called the ***quotient***.

So, division basically aims at finding out how many groups of one number fit into another number. The same approach applies when dividing a fraction by another fraction.

DIVISION OF A FRACTION BY ANOTHER FRACTION

Take a rectangular strip of paper, and divide it into seven parts. Shade five parts out of these seven parts. This shaded part represents $\frac{5}{7}$.

Now, each of the shaded parts represents $\frac{1}{7}$.



Count the number of sets of $\frac{1}{7}$ that fit into $\frac{5}{7}$. There are five shaded sets of $\frac{1}{7}$ that fit into $\frac{5}{7}$ of the whole strip.

Hence, we can say that there are five sets of $\frac{1}{7}$ in $\frac{5}{7}$.

Therefore, we can say that $\frac{5}{7} \div \frac{1}{7} = 5$.

So, $\frac{5}{7} \div \frac{1}{7}$ simply means finding how many times $\frac{1}{7}$ fits into $\frac{5}{7}$.

Divide $\frac{5}{6}$ *by* $\frac{5}{12}$ by the same method as above.

To do so, we have to find how many $\frac{5}{12}$ fit into $\frac{5}{6}$.

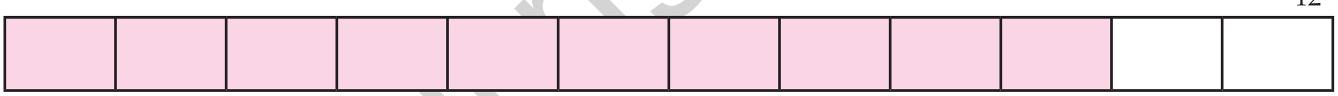
First, we have to convert them into like fractions.

$$\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$$

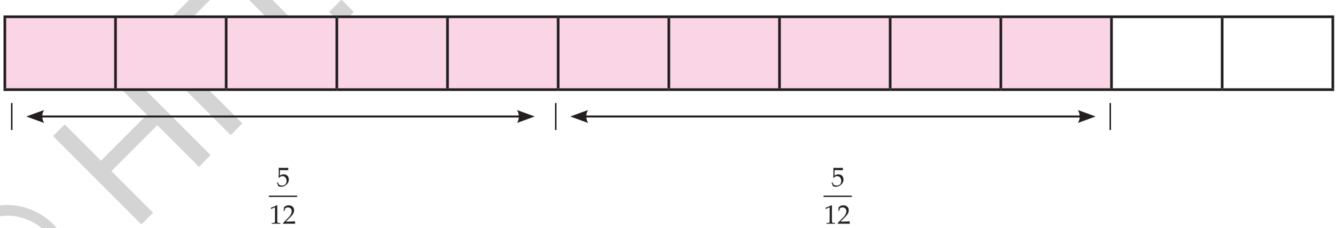
So, we have to find $\frac{10}{12} \div \frac{5}{12}$. Let us find the answer using following steps:

Step 1: Draw a rectangular strip, and divide it into 12 parts (equal to the denominator).

Step 2: Now shade 10 parts (numerator) out of these 12 parts. These coloured parts represent $\frac{10}{12}$.



Step 3: Mark the sets of $\frac{5}{12}$ that fit into the coloured parts that represent $\frac{10}{12}$. As each part is $\frac{1}{12}$, five parts represent $\frac{5}{12}$.



Step 4: Count the number of sets of $\frac{5}{12}$ that fit into $\frac{10}{12}$. We can see here that there are two sets of $\frac{5}{12}$ that fit into $\frac{10}{12}$. The result is the quotient.

So, $\frac{10}{12} \div \frac{5}{12} = 2$

Multiplicative Inverse (Reciprocal)

When a number is multiplied by its *multiplicative inverse*, the product is 1.

$$\frac{3}{2} \times \frac{2}{3} = \frac{6}{6} = 1$$

Reciprocal of an improper fraction is a proper fraction.

If $\frac{7}{3}$ is an improper fraction, its reciprocal is $\frac{3}{7}$.

To find the reciprocal of a fraction just interchange the numerator and the denominator.

Now let us reconsider the division of $\frac{5}{6}$ by $\frac{5}{12}$. Carefully observing the division process, you will find that $\frac{5}{6} \div \frac{5}{12} = \frac{5}{6} \times \frac{12}{5}$.

Just take the reciprocal of the second fraction and multiply. That is, just multiply the dividend fraction by the reciprocal of the divisor fraction.

$= \frac{\cancel{5}}{\cancel{6}} \times \frac{\cancel{12}^{2}}{\cancel{5}} = 2$ (on cancelling the common terms)

Hence, the simple rule for the division of a fraction by a fraction is as follows:

To divide a fraction by another fraction, we multiply the dividend fraction by the reciprocal of the divisor fraction.

That is, $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times$ reciprocal of $\frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$.

DIVISION OF A WHOLE NUMBER BY A FRACTION

Note the following steps to divide a whole number by a fraction:

First, replace ÷ sign with × sign.

Second, write the reciprocal of the divisor fraction. (If the fraction is a mixed fraction, first convert it into an improper fraction.) The whole number is the dividend fraction.

Third, multiply the numerator by the numerator and the denominator by the denominator.

We can write a whole number as a fraction; for example, 4 can be written $\frac{4}{1}$, 7 can be written $\frac{7}{1}$ and so on.

Let us divide $8 \div \frac{4}{5}$.

$= 8 \times \frac{5}{4} = {}^{2}\cancel{8} \times \frac{5}{\cancel{4}} = 2 \times 5 = 10$

The product of the whole number by the reciprocal of the fraction will be the quotient.

DIVISION OF A FRACTION BY A WHOLE NUMBER

When a fraction is divided by a whole number, the same rule of dividing a whole number by a fraction is applied. Here, the given fraction is the dividend and whole number is the divisor.

The reciprocals of whole numbers such as 4 and 7 are $\frac{1}{4}$ and $\frac{1}{7}$, respectively.

Let us divide $\frac{3}{7}$ by 9.

First, write 9 as a fraction, that is, $\frac{9}{1}$.

Second, find the reciprocal of 9, that is, $\frac{1}{9}$.

Third, multiply the fraction by the reciprocal of the whole number, that is,

$$= \frac{\cancel{3}}{7} \times \frac{1}{\cancel{9}_3}$$

$$= \frac{1}{21}$$

Cancel out the common factors from the numerators and the denominators of both the fractions that are multiplied.

Fourth, reduce the fraction to the lowest term.

As $\frac{1}{21}$ cannot be reduced further, so it is already in lowest terms.

SOME EXAMPLES

Example 1: Smriti has $8\frac{3}{4}$ kg of cotton wool for making cushions. If $1\frac{1}{4}$ kg of cotton is used to make one cushion, how many cushions can be made using the whole amount of cotton?

Solution: Total amount of cotton Smriti has = $8\frac{3}{4}$ kg = $\frac{35}{4}$ kg

Amount used to make one cushion = $1\frac{1}{4}$ kg = $\frac{5}{4}$ kg.

Number of cushions that can be made = $\frac{35}{4} \div \frac{5}{4} = \frac{35}{4} \times \frac{4}{5} = 7$.

So, Smriti can make seven cushions by using the total amount of cotton wool that she has.

Example 2: Amit can walk $2\frac{1}{3}$ km in an hour. How long will it take him to walk to the metro station that is 5 km from his house?

Solution: Time taken by Amit to walk a distance of $2\frac{1}{3}$ km = $\frac{7}{3}$ km is 1 hour.

Time taken by Amit to walk a distance of 1 km = $1 \div \frac{7}{3} = 1 \times \frac{3}{7} = \frac{3}{7}$ hours.

Time taken by Amit to walk a distance of 5 km = $5 \times \frac{3}{7} = \frac{5 \times 3}{7} = \frac{15}{7} = 2\frac{1}{7}$ hours.

Hence, time taken by Amit to reach the metro station is $2\frac{1}{7}$ hours.

Example 3: Ripu has ₹2175. This is $\frac{3}{8}$ of the amount she earned in a month. How much money did she earn?

Solution: Amount of money Ripu has = ₹2175 = $\frac{3}{8}$ × Amount of money she earned in a month.

So, amount of money she earned in a month = $2175 \div \frac{3}{8} = 2175 \times \frac{8}{3} = \frac{2175 \times 8}{3} = 725 \times 8$ = ₹5800.

Hence, Ripu earned ₹5800 per month.

Example 4: It takes 21 full specific types of trees to make 1 ton of paper. If there are 273 such trees in a forest then to save $\frac{7}{13}$ part of the forest, how much of paper do we have to save?

Solution: Amount of paper 21 trees can produce = 1 ton.

Amount of paper one tree can produce = $\frac{1}{21}$ tonnes.

Amount of paper 273 trees can produce = $273 \times \frac{1}{21}$ tonnes = $\frac{273}{21}$ tonnes.

Fraction of forest to be saved = $\frac{7}{13}$ of total trees.

Paper to be saved = $\frac{7}{13} \times \frac{273}{21} = \frac{273}{13 \times 3} = \frac{21}{3} = 7$ tonnes.

The amount of paper that can be saved is 7 tonnes.

PRACTICE EXERCISE 3.3

(1) Find the reciprocal for each of the following. Classify the reciprocals into proper fractions, improper fractions and whole numbers.

(i) $\frac{7}{17}$ (ii) $\frac{9}{23}$ (iii) $\frac{5}{6}$ (iv) 56

(v) $\frac{1}{18}$ (vi) $2\frac{3}{5}$ (vii) $\frac{9}{8}$ (viii) $\frac{12}{11}$

(2) Divide the following:

(i) 21 by $\frac{7}{12}$ (ii) 32 by $\frac{8}{14}$ (iii) 6 by $\frac{24}{13}$ (iv) 33 by $\frac{22}{9}$

(v) 40 by $3\frac{1}{3}$ (vi) $\frac{2}{5}$ by 4 (vii) $\frac{25}{9}$ by 5 (viii) $\frac{54}{45}$ by 9

(ix) $2\frac{6}{11}$ by 14 (x) $\frac{64}{77}$ by $\frac{1}{7}$ (xi) $\frac{6}{7}$ by $\frac{4}{5}$ (xii) $\frac{12}{18}$ by $\frac{13}{9}$

(xiii) $\frac{11}{2}$ by $\frac{1}{4}$ (xiv) $\frac{19}{21}$ by $\frac{3}{7}$ (xv) $\frac{48}{5}$ by $\frac{12}{25}$ (xvi) $\frac{23}{4}$ by $\frac{5}{3}$

(xvii) $6\frac{2}{5}$ by $2\frac{2}{3}$ (xviii) $\frac{49}{6}$ by $\frac{7}{12}$ (xix) $\frac{64}{21}$ by $\frac{8}{7}$ (xx) $12\frac{4}{7}$ by $4\frac{1}{3}$

(3) Simplify the following:

(i) $\frac{3}{5} \div \left(\frac{1}{6} \times \frac{1}{3}\right)$ (ii) $\frac{5}{7} \div \frac{11}{7} \div \frac{15}{13}$ (iii) $\frac{7}{5}$ of $\left(\frac{3}{4} + \frac{1}{2}\right) \div 3\frac{2}{4}$

(iv) $\left(\frac{4}{5} \text{ of } \frac{10}{6}\right) + \left(\frac{4}{3} - \frac{5}{3} \div \frac{5}{2}\right)$ (v) $\frac{13}{5} - \left(\frac{17}{5} \text{ of } \frac{10}{68}\right) \div \frac{1}{4}$

(4) The product of two numbers is $16\frac{5}{6}$. If one of the numbers is $10\frac{2}{3}$, find the other number.

(5) By what number should $13\frac{1}{3}$ be multiplied to get 64?

(6) The area of a rectangular park is $25\frac{1}{5}$ sq m. If the length of the park is $15\frac{1}{3}$ m, find its breadth.

(7) Nida covers a distance of $18\frac{2}{4}$ km in $7\frac{2}{5}$ hours on foot. How many kilometres does she walk per hour?

(8) If the cost of 1 L crude oil is ₹$78\frac{4}{5}$, how much of crude oil can be bought for ₹$225\frac{1}{7}$?

(9) In a refugee camp, 32 kg rice is distributed. If each refugee gets $\frac{4}{5}$ kg rice, how many refugees are there?

(10) By what number should $9\frac{1}{6}$ be divided to obtain $18\frac{1}{3}$?

(11) Guavas are sold at ₹$12\frac{1}{4}$ per kg. Find the weight of guavas that can be purchased for ₹$22\frac{2}{5}$?

(12) A drum contains $48\frac{3}{4}$ L paint. A small jug has a capacity of $\frac{1}{4}$ L. To empty the drum, how many jugs of the paint are required to be taken out of the drum?

PROJECT WORK

Collect wrappers of eatables, say namkeen, wafers, biscuits, chocolates etc. and note down the Energy (in kcal) given at the back under the nutrition facts. It is given in kcal per 100 g. Note it down in a table for each of the wrappers and calculate the amount of energy (in kcal) for $\frac{1}{2}$, $\frac{1}{5}$ th and $\frac{1}{10}$ th of 100 g for each of the wrappers collected.

MULTIPLE CHOICE QUESTIONS

(1) Which letter comes $\frac{2}{3}$ of the way from A and R?

(a) L (b) Q (c) M (d) J

(2) $\frac{1}{6}$ of a number equals $\frac{7}{9} \div \frac{1}{27}$. What is the number?

(a) $\frac{2}{2}$ (b) 18 (c) 21 (d) 126

(3) How much gift paper will be used to cover four gifts if each gift requires $2\frac{2}{5}$ m of paper, which also includes $\frac{1}{10}$ m of paper which is wasted in cutting and finishing?

(a) $4\frac{3}{5}$ m (b) $9\frac{1}{5}$ m (c) $8\frac{1}{10}$ m (d) $10\frac{4}{5}$ m

(4) Product of $\frac{3}{5} \times \frac{5}{7} \times \frac{7}{8} \times 8$ is equal to

(a) 3 (b) 2 (c) 7 (d) $\frac{3}{8}$

(5) The reciprocal of $1\frac{3}{11}$ is

(a) $\frac{11}{3}$ (b) $\frac{1}{11}$ (c) $\frac{11}{14}$ (d) $3\frac{1}{11}$

COMPREHENSIVE EXERCISE

(1) Simplify:

(i) $7\frac{1}{3} \times 2\frac{2}{11} - 5\frac{1}{4} \times \frac{3}{7}$ (ii) $\left(\frac{3}{8} \text{ of } \frac{24}{5}\right) + \left(\frac{6}{5} - \frac{3}{5} \div \frac{3}{2}\right)$ (iii) $\frac{1}{2\frac{2}{3}} + \frac{1}{3\frac{1}{5}} + \frac{1}{4\frac{4}{7}}$

(2) How many $\frac{1}{15}$ kg of candy boxes can be made with $2\frac{1}{3}$ kg of candies?

(3) A vegetable seller had $12\frac{1}{2}$ kg green chillies. He sold $\frac{2}{5}$ kg chillies to each of his four customers. He gave $\frac{4}{10}$ kg to another customer and gave $1\frac{1}{2}$ kg to his brother. How much green chillies were left with him?

(4) Rita bought a door of size 6 m $\times$ $8\frac{2}{5}$ m. Size of the door of her room is of dimensions $5\frac{1}{5}$ m $\times$ $7\frac{1}{5}$ m. How much area of the door should be cut off to fit the door of her room?

* For more practice questions refer to practice book.

CHAPTER CHECK-UP

- The product of two fractions is obtained by multiplying their numerators and denominators separately and by writing the product as $\frac{\text{Product of numerators}}{\text{Product of denominators}}$.
- To multiply a fraction with a whole number, just convert the whole number into a fraction by taking its denominator 1 and applying the same rule.
- The reciprocal of a fraction is obtained by interchanging the numerator and the denominator. While dividing one fraction by another fraction; we multiply the first fraction by the reciprocal of the other.

WEBLINKS:

http://www.mathsisfun.com/fractions-menu.html

http://www.aaamath.com/fra.htm